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NONLINEARITY OF ARCH AND STOCHASTIC VOLATILITY MODELS AND BARTLETT'S FORMULA

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Abstract: We review some notions of linearity of time series and show that ARCH or stochastic volatility (SV) processes are not only non-linear: they are not even weakly linear, i.e., they do not even have a martingale representation. Consequently, the use of Bartlett's formula is unwarranted in the context of data typically modeled as ARCH or SV processes such as financial returns. More surprisingly, we show that even the squares of an ARCH or SV process are not weakly linear. Finally, we discuss an alternative estimator for the variance of sample autocorrelations that is applicable (and consistent) in the context of financial returns data.

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