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UPPER AND LOWER CLASS SEPARATING SEQUENCES FOR BROWNIAN MOTION WITH RANDOM ARGUMENT

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Abstract: Let $\mathbf{X} = X_1, X_2, \ldots$ be a sequence of random variables, let W be a Brownian motion independent of $\mathbf{X}$ and let $Z_k = W(X_k)$. A numerical sequence (t_k) will be called an *upper* (*lower*) *class sequence* for $\{Z_k\}$ if

$$P(Z_k > t_k \text{ for infinitely many } k) = 0 \text{ (or 1, respectively).}$$

At a first look one might be tempted to believe that a "separating line" (t_k^0), say, between the upper and lower class sequences for $\{Z_k\}$ is directly related to the corresponding counterpart (s_k^0) for the process $\{X_k\}$. For example, by using the law of the iterated logarithm for the Wiener process a functional relationship

$$t_k^0 = \sqrt{2 s_k^0 \log\log s_k^0} \tag{0.1}$$

seems to be natural. If $X_k = |W_2(k)|$ for a second Brownian motion W_2 then we are dealing with an iterated Brownian motion, and it is known that the multiplicative constant $\sqrt{2}$ in (0.1) needs to be replaced by $2 \cdot 3^{-3/4}$, contradicting this simple argument.

We will study this phenomenon from a different angle by letting $\{X_k\}$ be an i.i.d. sequence. It turns out that the relationship between the separating sequences (s_k^0) and (t_k^0) in the above sense depends in an interesting way on the extreme value behavior of $\{X_k\}$.

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