HERZ–SCHUR MULTIPLIERS AND NON-UNIFORMLY BOUNDED REPRESENTATIONS OF LOCALLY COMPACT GROUPS

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Abstract: Let G be a second countable, locally compact group and let φ be a continuous Herz–Schur multiplier on G. Our main result gives the existence of a (not necessarily uniformly bounded) strongly continuous representation π of G on a Hilbert space $\mathscr{H}$, together with vectors $\xi, \eta \in \mathscr{H}$, such that $\varphi(y^{-1}x) = \langle \pi(x)\xi, \pi(y^{-1})^*\eta \rangle$ for $x, y \in G$ and $\sup_{x \in G} \|\pi(x)\xi\| \cdot \sup_{y \in G} \|\pi(y^{-1})^*\eta\| = \|\varphi\|_{M_0A(G)}$. Moreover, we obtain control over the growth of the representation in the sense that $\|\pi(g)\| \leq \exp\left(\frac{c}{2}\mathrm{d}(g, e)\right)$ for $g \in G$, where $e \in G$ is the identity element, c is a constant, and d is a metric on G.

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