PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 34, Fasc. 2 (2014), pp. 293–316

PERSISTENCE OF SOME ITERATED PROCESSES

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Abstract: We study the asymptotic behaviour of the probability that a stochastic process $(Z_t)_{t\geq 0}$ does not exceed a constant barrier up to time T (a so-called *persistence probability*) when Z is the composition of two independent processes $(X_t)_{t\in I}$ and $(Y_t)_{t\geq 0}$. To be precise, we consider $(Z_t)_{t\geq 0}$ defined by $Z_t = X \circ |Y_t|$ if $I = [0,\infty)$ and $Z_t = X \circ Y_t$ if $I = \mathbb{R}$.

For continuous self-similar processes $(Y_t)_{t\geq 0}$, the rate of decay of persistence probability for Z can be inferred directly from the persistence probability of X and the index of self-similarity of Y. As a corollary, we infer that the persistence probability for iterated Brownian motion decays asymptotically like $T^{-1/2}$.

If Y is discontinuous, the range of Y possibly contains gaps, which complicates the estimation of the persistence probability. We determine the polynomial rate of decay for X being a Lévy process (possibly two-sided if $I = \mathbb{R}$) or a fractional Brownian motion and Y being a Lévy process or random walk under suitable moment conditions.

2000 AMS Mathematics Subject Classification: Primary: 60G99; Secondary: 60G18, 60G50, 60G51, 60J65.

Keywords and phrases: Iterated process, one-sided barrier problem, one-sided exit problem, persistence, persistence probability, small deviation probability, survival probability.