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CAPTIVITY OF MEAN-FIELD PARTICLE SYSTEMS AND THE RELATED EXIT PROBLEMS

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Abstract: A mean-field system is a weakly interacting system of N particles in $\mathbb{R}^d$ confined by an external potential. The aim of this work is to establish a simple result about the exit problem of mean-field systems from some domains when the number of particles goes to infinity. More precisely, we prove the existence of some subsets of $\mathbb{R}^{dN}$ such that the probability of leaving these sets before any $T > 0$ is arbitrarily small by taking N large enough. On the one hand, we show that the number of steady states in the small-noise limit is arbitrarily large with a sufficiently large number of particles. On the other hand, using the long-time convergence of the hydrodynamical limit, we identify the steady states as N goes to infinity with the invariant probabilities of the McKean–Vlasov diffusion so that some steady states in the small-noise limit are not steady states in the large N limit.

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