PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 35, Fasc. 2 (2015), pp. 191–200

HCM PROPERTY AND THE HALF-CAUCHY DISTRIBUTION

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Abstract: Let Z_α and $\tilde{Z}_\alpha$ be two independent positive α-stable random variables. It is known that $(Z_\alpha/\tilde{Z}_\alpha)^\alpha$ is distributed as the positive branch of a Cauchy random variable with drift. We show that the density of the power transformation $(Z_\alpha/\tilde{Z}_\alpha)^\beta$ is hyperbolically completely monotone in the sense of Thorin and Bondesson if and only if $\alpha \leq 1/2$ and $|\beta| \geq \alpha/(1-\alpha)$. This clarifies a conjecture of Bondesson (1992) on positive stable densities.

2000 AMS Mathematics Subject Classification: Primary: 60E07; Secondary: 60E05.

Keywords and phrases: Half-Cauchy distribution, complete monotonicity, generalized gamma convolution, hyperbolically completely monotone function, Pick function, positive stable density.

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