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FREE INFINITE DIVISIBILITY FOR GENERALIZED POWER DISTRIBUTIONS WITH FREE POISSON TERM*

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Abstract. We study free infinite divisibility (FID) for a class of generalized power distributions with free Poisson term by using complex analytic methods and free cumulants. In particular, we prove that (i) if X follows the free generalized inverse Gaussian distribution, then the distribution of X^r is FID when $|r| \geqslant 1$; (ii) if S follows the standard semicircle law and $u > 2$, then the distribution of $(S+u)^r$ is FID when $r \leqslant -1$; (iii) if B_p follows the beta distribution with parameters p and $3/2$, then (iii-a) the distribution of B_p^r is FID when $|r| \geqslant 1$ and $0 < p \leqslant 1/2$; (iii-b) the distribution of B_p^r is FID when $r \leqslant -1$ and $p > 1/2$.

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