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ON STRONGLY ORTHOGONAL MARTINGALES IN UMD BANACH SPACES

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Abstract. In the present paper we introduce the notion of strongly orthogonal martingales. Moreover, we show that for any UMD Banach space X and for any X-valued strongly orthogonal martingales M and N such that N is weakly differentially subordinate to M, one has, for all $1 < p < \infty$,

$$\mathbb{E}\|N_t\|^p \leqslant \chi_{p,X}^p \mathbb{E}\|M_t\|^p, \quad t \geqslant 0,$$

with the sharp constant $\chi_{p,X}$ being the norm of a decoupling-type martingale transform and lying in the range

$$\begin{aligned}\max\{\sqrt{\beta_{p,X}}, \sqrt{\hbar_{p,X}}\} &\leqslant \max\{\beta_{p,X}^{\gamma,+}, \beta_{p,X}^{\gamma,-}\} \\ &\leqslant \chi_{p,X} \leqslant \min\{\beta_{p,X}, \hbar_{p,X}\},\end{aligned}$$

where $\beta_{p,X}$ is the UMD_p constant of X, $\hbar_{p,X}$ is the norm of the Hilbert transform on $L^p(\mathbb{R}; X)$, and $\beta_{p,X}^{\gamma,+}$ and $\beta_{p,X}^{\gamma,-}$ are the Gaussian decoupling constants.

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Key words and phrases: strongly orthogonal martingales, weak differential subordination, UMD, sharp estimates, decoupling constant, martingale transform, Hilbert transform, diagonally plurisubharmonic function.

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