EXTREMES OF ORDER STATISTICS OF STATIONARY GAUSSIAN PROCESSES

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Abstract: Let $\{X_i(t), t \geqslant 0\}$, $1 \leqslant i \leqslant n$, be mutually independent and identically distributed centered stationary Gaussian processes. Under some mild assumptions on the covariance function, we derive an asymptotic expansion of

$$\mathbb{P}\big(\sup_{t \in [0, xm_r(u)]} X_{(r)}(t) \leqslant u\big) \quad \text{as } u \to \infty,$$

where

$$m_r(u) = \big(\mathbb{P}(\sup_{t \in [0,1]} X_{(r)}(t) > u)\big)^{-1}\big(1 + o(1)\big),$$

and $\{X_{(r)}(t), t \geqslant 0\}$ is the rth order statistic process of $\{X_i(t), t \geqslant 0\}$, $1 \leqslant i, r \leqslant n$. As an application of the derived result, we analyze the asymptotics of supremum of the order statistic process of stationary Gaussian processes over random intervals.

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