EMBEDDED MARKOV CHAIN APPROXIMATIONS IN SKOROKHOD TOPOLOGIES

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Abstract: We prove a J_1-tightness condition for embedded Markov chains and discuss four Skorokhod topologies in a unified manner.

To approximate a continuous time stochastic process by discrete time Markov chains, one has several options to embed the Markov chains into continuous time processes. On the one hand, there is a Markov embedding which uses exponential waiting times. On the other hand, each Skorokhod topology naturally suggests a certain embedding. These are the step function embedding for J_1, the linear interpolation embedding for M_1, the multistep embedding for J_2 and a more general embedding for M_2. We show that the convergence of the step function embedding in J_1 implies the convergence of the other embeddings in the corresponding topologies. For the converse statement, a J_1-tightness condition for embedded time-homogeneous Markov chains is given.

Additionally, it is shown that J_1 convergence is equivalent to the joint convergence in M_1 and J_2.

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