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QUENCHED ASYMPTOTICS FOR SYMMETRIC LÉVY PROCESSES INTERACTING WITH POISSONIAN FIELDS

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Abstract. We establish the quenched large time asymptotics for the Feynman–Kac functional

$$\mathbb{E}_x\Big[\exp\Big(-\int_0^t V^\omega(Z_s)\,ds\Big)\Big]$$

associated with a pure-jump symmetric Lévy process $(Z_t)_{t\geqslant 0}$ in general Poissonian random potentials V^ω on $\mathbb{R}^d$, which is closely related to the large time asymptotic behavior of solutions to the nonlocal parabolic Anderson problem with Poissonian interaction. In particular, when the density function with respect to the Lebesgue measure of the associated Lévy measure is given by

$$\rho(z) = \frac{1}{|z|^{d+\alpha}}\mathbb{1}_{\{|z|\leqslant 1\}} + e^{-c|z|^\theta}\mathbb{1}_{\{|z|>1\}}$$

for some $\alpha \in (0,2)$, $\theta \in (0,\infty]$ and $c > 0$, an explicit quenched asymptotics is derived for potentials with the shape function given by $\varphi(x) = 1 \wedge |x|^{-d-\beta}$ for $\beta \in (0,\infty]$ with $\beta \neq 2$, and it is completely different for $\beta > 2$ and $\beta < 2$. We also discuss the quenched asymptotics in the critical case (e.g., $\beta = 2$ in the example above). The work fills the gaps of the related work for pure-jump symmetric Lévy processes in Poissonian potentials, where only the case that the shape function is compactly supported (e.g., $\beta = \infty$ in the example above) has been handled in the literature.

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