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## A REMARK ON THE EXACT LAWS OF LARGE NUMBERS FOR RATIOS OF INDEPENDENT RANDOM VARIABLES

BY

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**Abstract.** Let  $(X_n)_{n\in\mathbb{N}}$  and  $(Y_n)_{n\in\mathbb{N}}$  be two sequences of i.i.d. random variables which are independent of each other and all have the distribution of a positive random variable  $\xi$  with density  $f_{\xi}$ . We study weighted strong laws of large numbers for the ratios of the form  $\frac{1}{b_n}\sum_{k=1}^n a_k \frac{X_k}{Y_k}$  in the cases when  $\mathbb{E}\xi=\infty$  or  $\lim_{x\to 0^+}f_{\xi}(x)=0$  or  $f_{\xi}$  is unbounded. This research complements some results known so far.

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