ABSTRACTS OF TALKS OF 11th WORKSHOP:
NON-COMMUTATIVE HARMONIC ANALYSIS
WITH APPLICATIONS TO PROBABILITY,
17 - 23.08.2008, Będlewo, Poland

1. **Michael Anshelevich (Texas A&M)**

Belinschi – Nica transformations and free convolution semigroups.

Abstract: In 2007, Belinschi and Nica described the relation between the following three objects. $\{\gamma_t\}$ are the semicircular distributions, which form a semigroup with respect to the free convolution $\boxplus$. Φ is a map on measures coming from the shift of their Jacobi coefficients. Finally, $\mathbb{B}_t$ is another family of maps on measures, constructed using the free convolution $\boxplus$ and Boolean convolution $\uplus$, which form a semigroup, and which also have the property that $\mathbb{B}_1$ maps all measures precisely onto the freely infinitely divisible ones.

Belinschi and Nica proved that for any measure ν,

$$\Phi\left[\nu \boxplus \gamma_t\right] = \mathbb{B}_t\left[\Phi\left[\nu\right]\right].$$

I will explain that this property is not special for the semicircular distributions. Namely, for any free convolution semigroup $\{\rho_t\}$ I will describe a family of naturally occurring maps Φ^ρ such that

$$\Phi^\rho\left[\nu \boxplus \rho_t\right] = \mathbb{B}_t\left[\Phi^\rho\left[\nu\right]\right].$$

The maps are constructed using an operator model, and the framework in which these maps arise is conditionally free probability. They are also related to a number of other operations in non-commutative probability.

2. **Andreas Boukas (Athens)**

The $*$–Lie Algebra of the Renormalized Higher Powers of White Noise (RHPWN) – joint work with Prof. Luigi Accardi.

Abstract: We present:

(i) The problem of giving meaning to the powers of the Dirac delta function in relation to the problem of giving meaning to the powers of the creation and annihilation densities.
(ii) The commutation relations of the $*$–Lie Algebra of the Renormalized Higher Powers of White Noise (RHPWN).
(iii) The connection between RHPWN and the centerless Virasoro (or Witt)-Zamolodchikov-w_∞ $*$–Lie algebras of conformal field theory.
(iv) The associated Fock space construction.
(v) The central extensions of the RHPWN and w_∞ $*$–Lie algebras.

3. **Marek Bożejko (Wrocław)**

Positive definite functions on Coxeter and free groups.

Abstract: We present the following subjects. Let G be a free group or Coxeter group (W, S), then

(a) The length functions are strongly negative definite on G, i.e. every operator valued quasi–multiplicative function on G is completely positive definite.
(b) Generalized Riesz product functions on G are positive definite and this implies that the operator–valued Khinchine inequality holds .

(c) Applications to operator spaces and noncommutative probability will be also done.

4. **Nizar Demni (Bielefeld)**

Topics on Meixner families – joint work with Marek Bożejko.

Abstract: Several, seemingly independent, approaches led to the Meixner family of orthogonal polynomials in both the classical and the free settings. We shed the light on how these approaches are inter-related and provide some new characterizations using the free derivative operator and free probability. We finally investigate the q-deformed setting which is rather more complicated than the particular cases $q = 0$ and $q = 1$.

5. **Jan Dereziński (Warszawa)**

Reduced and extended weak coupling limit.

Abstract: The weak coupling limit is the study of dynamics when the coupling constant λ goes to zero and time is rescaled as t/λ^2. Apart from the relatively well known weak coupling limit for reduced dynamics I will discuss "extended weak coupling limit", which does not involve a reduction and goes back to Accardi, Frigerio and Lu, who called it "stochastic limit". I will describe a new version of this idea based on the recent work of W. DeRoeck and myself.

The main motivation for considering these constructions comes from the physics of open quantum systems and is related to theory of quantum measurements. They are also very interesting from the point of view of pure mathematics.

6. **Franco Fagnola (Milano)**

Structure of generators of symmetric quantum Markov semigroups.

Abstract: A quantum Markov semigroup $\mathcal{T}$ on $\mathcal{B}(\mathsf{h})$ with a faithful normal invariant state ρ admits a dual $\mathcal{T}'$ satisfying $\mathrm{tr}(\rho^{1/2}x\rho^{1/2}\mathcal{T}_t(y)) = \mathrm{tr}(\rho^{1/2}\mathcal{T}_t'(x)\rho^{1/2}y)$ for all $x, y \in \mathcal{B}(\mathsf{h})$. In this talk we discuss the characterisation of the generators $\mathcal{L}(x) = G^*x + \sum_\ell L_\ell^* x L_\ell + xG$ of norm continuous quantum Markov semigroups that are symmetric (i.e. $\mathcal{T} = \mathcal{T}'$) or satisfy some quantum detailed balance condition. In particular we show that $\mathcal{T}$ is symmetric if and only if an intertwining relationship on $\rho^{1/2}$, the L_ℓ's and their adjoints holds.

We also discuss the same problems when $\mathcal{T}$ is defined by other dualities.

References

[1] F. Fagnola, V. Umanit , *Generators of detailed balance quantum Markov semigroups,* IDAQP, **10** no.3, 335–363 (2007)

[2] F. Fagnola, V. Umanit , *Detailed balance, time reversal and generators of quantum Markov semigroups,* To appear in Math. Notes

7. **Maxime Février (Toulouse)**

The Free Multiplicative Convolution Semigroup Of the Free Poisson Distribution.

Abstract: We provide some information on the free multiplicative convolution semigroup of the famous Marchenko-Pastur distribution. We also prove that one of these interpretations can be

generalized to any compactly supported probability measure on the half-line, in a try to obtain a result analogous to the work of Nica and Speicher in the additive case.

8. **Jan Florek (Wrocław)**

Billiard, diophantine approximation and related problems.

Abstract: "The $3g$–gap theorem"
Let $0 < q < 1$ and T be the rotation of $[0, 1)$ by q. If M is the union of g disjoint open intervals contained in $(0, 1)$, the induced automorphism T_M is an interval exchange of at most $3g$ intervals.

9. **Uwe Franz (Besan on)**

The Meixner classes for Free and Monotone Independence.

Abstract: Bożejko and Bryc have shown that quadratic regression for free random variables can be used to characterize the free Meixner classes. In my talk, I will study quadratic regression for monotone independence, and the relation between the free and the monotone case.

10. **Kei Harada (Nagoya)**

Semigroups of Wiener Processes on Abstract Wiener Spaces.

Abstract: We prove the semigroup of Wiener process P_t on an abstract Wiener space X is locally equicontinuous on $C_b(X)$, equipped with T_t-topology introduced by L. Le Cam. We obtain a "Laplacian" as its generator. Applications to the analysis of measures and the Dirichlet problem are discussed.

11. **Fumio Hiai (Tohoku)**

Pressure and its Legendre transform in microstate free entropy.

Abstract: We previously introduced the notion of free pressure and defined the free entropy-like quantity (called eta-entropy) for noncommutative random variables [Comm. Math. Phys. 255 (2005), 229-252]. With T. Miyamoto and Y. Ueda we further proposed the orbital approach to the microstate free entropy and free entropy dimension [Internat. J. Math., to appear]. This talk is concerned with the orbital versions of free pressure and eta-entropy. After defining these quantities and giving their basic properties we examine the relations of the orbital eta-entropy with the previous eta-entropy and the orbital free entropy. The zero orbital eta-entropy case is characterized based on a certain free transportation cost inequality. Furthermore, the case of R-diagonal random variables is exemplified in some detail.

12. **Robin Hudson (Loughborough)**

Causal and rectangular double products in quantum stochastic calculus.

Abstract: Double products of rectangular and causal (triangular) form will be defined as solutions of quantum stochastic differential equations in two different but equivalent ways. Some examples can be constructed explicitly as implementors of Bogolubov transformations. Causal products in a non-Fock calculus can be used to formulate a rotational analog of the usual (translational) Cameron-Martin-Girsanov theorem and when combined with a corresponding martingale representation theorem, they suggest a quantum version of the Black-Scholes model in financial mathematics.

13. **Un Cig Ji (Chungbuk)**

Regular Properties of Quantum Stochastic Gradients and Quantum Stochastic Integrals.

Abstract: Motivated by the quantum white noise theory we introduce new notions of *quantum stochastic gradients* (creation, annihilation and conservation gradients) of operators on the Fock space which are quantum extensions of the classical stochastic gradient. Then by the adjoint actions of the quantum stochastic gradients we define (non-adapted) quantum stochastic integrals of Hitsuda–Skorohod type and so called the *Hitsuda–Skorohod quantum stochastic integrals*. We study regular properties of the Hitsuda–Skorohod quantum stochastic integrals in terms of regularities of quantum stochastic gradients.

14. **Hiroaki Kakuma (Nagoya)**

Fourier Transform on Infinite Dimensional Spaces.

Abstract: Fourier transform of L^2 space on the Euclidean space plays a great part in analysis due to Parseval's theorem, or compatibility with differentiation. Our purpose is to formulate a Fourier-like transform on infinite dimensional spaces. In particular, we construct a unitary transform similar to the Fourier transform in case of $\mathbb{R}^\infty$. We emphasize that the domain of this transform is a Hilbert space corresponding to L^2 space on $\mathbb{R}^\infty$. Although there is not Lebesgue measure on an infinite dimensional space, we can construct such a space by making use of measure theory on infinite dimensional spaces.

15. **Dorota Kępa (Lublin)**

Bassalygo-Dobrushin uniqueness for continuous spin systems on quasi-bounded graphs.

Abstract: We present an extension of the Bassalygo-Dobrushin technique of proving uniqueness of Gibbs fields on quasi-bounded graphs, developed in [Theory of Probab. Appl. 31, 572-589 (1986)], to the case of continuous spins.

16. **Anna Kula (Kraków)**

q-normality, q-positive definiteness and related convolutions.

Abstract: Motivated by the notion of q-normal operators (those which satisfy the relation $NN^* = qN^*N$ for a $q > 0$), we define q-deformations of positivity and complete monotonicity, and present the related integral representation. Next, we study unital $*$-algebras generated by families of elements, which are q-normal and satisfy some additional commutation relations. A realization of such elements can be found in the $*$-algebra of operators on a pre-Hilbert space. The distribution of sum of such operators, with respect to a given state, gives rise to a formula which can be interpreted as a new convolution of measures. We investigate its positivity preserving properties and also present a non-commutative analogue of the classical central limit theorem. A part of the results is a joint work with Éric Ricard.

17. **Franz Lehner (Graz)**

Eigenfunctions of lamplighter random walks and percolation clusters on graphs.

Abstract: Continuing last year's talk, we show that the Plancherel measure of the lamplighter random walk on a graph coincides with the expected spectral measure of the absorbing random walk on the Bernoulli percolation clusters. In the subcritical regime the spectrum is pure point and we construct a complete orthonormal basis consisting of finitely supported eigenfunctions.

18. **Michael Leinert (Heidelberg)**

On translation of positive definite functions (with Lars Omlor).

Abstract: On IN-groups (groups with a compact invariant neighbourhood of the identity), translates of positive definite functions satisfy an inequality which is related to a theorem of Wiener.

19. **Romuald Lenczewski (Wrocław)**

A new model of noncommutative probability related to free probability.

Abstract: I will present some results on a new model of noncommutative probability related to free probability. In particular, I will discuss (some or all of) the following topics: concept of independence, product of Hilbert spaces, product of states, Fock space, central limit theorems, continued fractions, decompositions of limit laws, realizations in terms of walks on trees and Catalan paths, realizations on Fock spaces, asymptotic independence and a connection with random matrices.

20. **Eugene Lytvynov (Swansea)**

Diffusion approximation for equilibrium Kawasaki dynamics in continuum.

Abstract: A Kawasaki dynamics in continuum is a dynamics of an infinite system of interacting particles in R^d which randomly hop over the space. In our talk, we will deal with an equilibrium Kawasaki dynamics which has a Gibbs measure μ as invariant measure. We study a diffusive limit of such a dynamics, derived through a scaling of both the jump rate and time. Under weak assumptions on the potential of pair interaction, ϕ, (in particular, admitting a singularity of ϕ at zero), we prove that, on a set of smooth local functions, the generator of the scaled dynamics converges to the generator of an equilibrium diffusive dynamics of an infinite system of interacting particles.If the set on which the generators converge is a core for the diffusion generator, the latter result implies the weak convergence of finite-dimensional distributions of the corresponding equilibrium processes. In particular, if the potential ϕ is sufficiently smooth and sufficiently quickly converges to zero at infinity, we conclude from a result by Choi, Park, Yoo that the convergence of processes holds when the limiting diffusion is the gradient stochastic dynamics.

21. **Wojtek Młotkowski (Wrocław)**

Fuss – Catalan numbers in noncommutative probability.

Abstract: We prove that the Fuss-Catalan sequence $\binom{mp+r}{m}\frac{r}{mp+r}$ is positive definite if either $p \geq 1$ and $0 \leq r \leq p$ or $p \leq 0$ and $p-1 \leq r \leq 0$. It generalizes a recent result by T. Banica, S. T. Belinschi, M. Captaine, B. Collins, who proved this in the case $r = 1$. We study the family

the of corresponding probability measures $\mu(p, r)$ from the point of view of noncommutative probability.

22. **Jonathan Novak (Kingston)**

Random contractions and a deformation of the increasing subsequence problem.

Abstract: There is an interesting one-parameter family of deformations of Dyson's "Circular Unitary Ensemble" (the unitary group under Haar measure), in which the unitary group is replaced by a matrix ball and Haar measure is deformed to its pushforward under the operation of "truncation" of matrices. These ensembles were introduced by Sommers and Zyczkowski in connection with models of chaotic scattering. We show that certain averages over such truncated ensembles are rational numbers which, with the right scaling, enumerate transitions between equilibrium states in a certain model of interacting particles on the integer lattice. When the deformation parameter q is zero, this reduces to the enumeration of permutations with bounded increasing subsequence length.

23. **Hiromichi Ohno (Kyushu)**

Free energy density for mean field perturbation of states of a one-dimensional spin chain.

Abstract: Motivated by recent developments of quantum large deviation theory in spin chains, we reconsider the work of Petz, Raggio and Verbeure in 1989 on the variational expression of free energy density for the perturbation of mean field type. We extend it from the product state case to the general state case which satisfies factorization properties.

A one-dimensional spin chain is described by the infinite tensor product C^*-algebra $\mathcal{A} := \bigotimes_{k\in\mathbb{Z}} \mathcal{A}_k$ of full matrix algebras $\mathcal{A}_k := M_d(\mathbb{C})$ over $\mathbb{Z}$. The right-translation automorphism of $\mathcal{A}$ is denoted by γ. We denote by $\mathcal{S}_\gamma(\mathcal{A})$ the set of all γ-invariant states of $\mathcal{A}$. The C^*-subalgebra of $\mathcal{A}$ corresponding to a subset X of $\mathbb{Z}$ is $\mathcal{A}_X := \bigotimes_{k\in X} \mathcal{A}_k$. If $X \subset Y \subset \mathbb{Z}$, then $\mathcal{A}_X \subset \mathcal{A}_Y$ by a natural inclusion. The local algebra is a dense $*$-subalgebra $\mathcal{A}_{\rm loc} := \bigcup_{n=1}^{\infty} \mathcal{A}_{[-n,n]}$ of $\mathcal{A}$. The self-adjoint parts of $\mathcal{A}_{\rm loc}$ and $\mathcal{A}$ are denoted by $\mathcal{A}_{\rm loc}^{\rm sa}$ and $\mathcal{A}^{\rm sa}$, respectively. The usual trace on $\mathcal{A}_X$ for each finite $X \subset \mathbb{Z}$ is denoted by Tr without referring to X since it causes no confusion.

An interaction Φ in $\mathcal{A}$ is a mapping from the finite subsets of $\mathbb{Z}$ into $\mathcal{A}$ such that $\Phi(\emptyset) = 0$ and $\Phi(X) = \Phi(X)^* \in \mathcal{A}_X$ for each finite $X \subset \mathbb{Z}$. Given an interaction Φ and a finite subset $\Lambda \subset \mathbb{Z}$, the *local Hamiltonian* $H_\Lambda(\Phi)$ is given by

$$H_\Lambda(\Phi) := \sum_{X\subset\Lambda} \Phi(X), \tag{1}$$

and the *surface energy* $W_\Lambda(\Phi)$ is defined by

$$W_\Lambda(\Phi) := \sum \{\Phi(X) : X \cap \Lambda \neq \emptyset,\ X \cap \Lambda^c \neq \emptyset\},$$

whenever the sum converges in norm. We always assume that Φ is γ-invariant, i.e., $\gamma(\Phi(X)) = \Phi(X + 1)$ for every finite $X \subset \mathbb{Z}$, where $X + 1 := \{k + 1 : k \in X\}$. We denote by $\mathcal{B}_0(\mathcal{A})$ the set of all γ-invariant interactions Φ in $\mathcal{A}$ such that

$$\|\Phi\|_0 := \sum_{X\ni 0} \|\Phi(X)\| + \sup_{n\geq 1} \|W_{[1,n]}(\Phi)\| < +\infty.$$

It is easy to see that $\mathcal{B}_0(\mathcal{A})$ is a real Banach space with the usual linear operations and the norm $\|\Phi\|_0$. Associated with $\Phi \in \mathcal{B}_0(\mathcal{A})$ we have a strongly continuous one-parameter automorphism group α^Φ of $\mathcal{A}$ given by

$$\alpha_t^\Phi(A) = \lim_{m\to-\infty, n\to\infty} e^{itH_{[m,n]}(\Phi)} A e^{-itH_{[m,n]}(\Phi)}, \qquad A \in \mathcal{A}.$$

Then it is known [Ar1] that there exists a unique α^Φ-*KMS state* (at $\beta = -1$) φ of $\mathcal{A}$, which is automatically faithful and extremally γ-invariant (i.e., an extremal point of $\mathcal{S}_\gamma(\mathcal{A})$). The KMS state φ is also characterized by the Gibbs condition as well as by the variational principle, and so φ is also called the (global) *Gibbs state* for Φ. Below let us briefly recall the variational principle for φ (see [BR2] for details).

For a state ω of $\mathcal{A}$ we write ω_n for the restriction of ω to $\mathcal{A}_{[1,n]}$, and $D(\omega_n)$ stands for the density matrix of ω_n satisfying $\omega_n(A) = \operatorname{Tr} D(\omega_n)A$ for all $A \in \mathcal{A}_{[1,n]}$. The *mean entropy* (or the *entropy density*) of $\omega \in \mathcal{S}_\gamma(\mathcal{A})$ can be given by

$$s(\omega) := \lim_{n\to\infty} \frac{1}{n} S(\omega_n),$$

where $S(\omega_n) := -\operatorname{Tr} D(\omega_n) \log D(\omega_n)$, the *von Neumann entropy* of ω_n. The *mean energy* A_Φ of $\Phi \in \mathcal{B}_0(\mathcal{A})$ is denoted by

$$A_\Phi := \sum_{X \ni 0} \frac{\Phi(X)}{|X|}. \tag{2}$$

The *pressure* (or the *free energy density*) of Φ can be given by

$$P(\Phi) := \lim_{n\to\infty} \frac{1}{n} \log \operatorname{Tr} e^{-H_n(\Phi)},$$

where we write $H_n(\Phi) := H_{[1,n]}(\Phi)$ for short. Then the variational expressions of $P(\Phi)$ and $s(\omega)$ are

$$P(\Phi) = \max\{-\omega(A_\Phi) + s(\omega) : \omega \in \mathcal{S}_\gamma(\mathcal{A})\}, \tag{3}$$

$$s(\omega) = \inf\{\omega(A_\Phi) + P(\Phi) : \Phi \in \mathcal{B}_0(\mathcal{A})\}. \tag{4}$$

We will consider extensions of a pressure and a variational expression when the interaction is perturbed by a mean field term

$$s_n(A) = \frac{1}{n} \sum_{i=0}^{n-l} \gamma^i(A)$$

for $A \in \mathcal{A}_{[1,l]}$.

Definition 1.1. *A state $\varphi \in S_\gamma(\mathcal{A})$ satisfies the upper factorization property if*

$$\varphi \le \alpha(\varphi|_{\mathcal{A}_{(-\infty,0]}}) \otimes (\varphi|_{\mathcal{A}_{[1,\infty)}})$$

for some $\alpha > 0$. Similarly, φ satisfies the lower factorization property if

$$\varphi \ge \beta(\varphi|_{\mathcal{A}_{(-\infty,0]}}) \otimes (\varphi|_{\mathcal{A}_{[1,\infty)}})$$

for some $\beta > 0$.

Gibbs states satisfy the upper and lower factorization properties and finitely correlated states satisfy the upper factorization property. But the lower factorization property for finitely correlated states is quite strong.

By using these factorization properties, we can prove the following Propositions.

Proposition 1.2. *If $\varphi \in S_\gamma(\mathcal{A})$ satisfies the upper factorization property, then the mean relative entropy*

$$S_M(\omega, \varphi) = \lim_{n\to\infty} \frac{1}{n} S(\omega_n, \varphi_n)$$

exists for any $\omega \in S_\gamma(\mathcal{A})$, where $S(\omega_n, \varphi_n) = \omega_n(\log D(\omega_n) - \log D(\varphi_n))$, the mean entropy of ω_n and φ_n. Moreover the function $\omega \mapsto S_M(\omega, \varphi)$ is affine and weakly lower semicontinuous on $S_\gamma(\mathcal{A})$.*

Proposition 1.3. *If $\varphi \in S_\gamma(\mathcal{A})$ satisfies the upper factorization property, then the pressure*

$$p_\varphi(A) = \lim_{n\to\infty} \frac{1}{n} \log \mathrm{Tr}\, \exp\left(\log D(\varphi_n) - ns_n(A)\right)$$

exists for any $A \in \mathcal{A}_{\mathrm{loc}}^{\mathrm{sa}}$. Moreover the function p_φ is Lipschitz continuous as

$$|p_\varphi(A) - p_\varphi(B)| \le \|A - B\|.$$

Proposition 1.4. *If $\varphi \in S_\gamma(\mathcal{A})$ satisfies the upper and lower factorization properties, then the variational expression*

$$p_\varphi(A) = \max\{-\omega(A) - S_M(\omega, \varphi) : \omega \in S_\gamma(\mathcal{A})\}$$

for any $A \in \mathcal{A}_{\mathrm{loc}}^{\mathrm{sa}}$ and

$$\begin{aligned} S_M(\omega, \varphi) &= \sup\{-\omega(A) - p_\varphi(A) : A \in \mathcal{A}_{\mathrm{loc}}^{\mathrm{sa}}\} \\ &= \sup\{-\omega(A) - p_\varphi(A) : A \in \mathcal{A}^{\mathrm{sa}}\} \end{aligned}$$

for any $\omega \in S_\gamma(\mathcal{A})$ holds.

References

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24. **Narutaka Ozawa (Tokyo)**

Recent advances of in classification of finite von Neumann algebras.

Abstract: In recent few years, the classification program of finite von Neumann algebras has seen a remarkable progress. I will review this subject and report some new results obtained by S. Popa and myself.

25. **Artur Płaneta (Kraków)**

Olson's order for selfadjoint operators in Hilbert space - joint work with Jan Stochel.

Abstract: Let A_1 and A_2 be selfadjoint operators in a Hilbert space H with spectral distributions F_1 and F_2, respectively. We denote $A_1 \preccurlyeq A_2$ if $F_2(x) \le F_1(x)$ for all $x \in \mathbb{R}$. Properties of this order are investigated.

26. **Adam Skalski (Lancaster & Łódź)**

On some questions connected with Voiculescu's noncommutative topological entropy – joint work with Joachim Zacharias.

Abstract: Noncommutative topological entropy is a numerical invariant associated to a given automorphism (or a completely positive map) of a nuclear (or exact) C^*–algebra. It was introduced in 1995 by Voiculescu and is closely related to C^*–algebraic approximation properties. We will recall basic facts concerning this notion and present two recent related results:

a) noncommutative entropy does not change under taking canonical extensions to crossed products of actions of discrete amenable quantum groups

b) we can obtain estimates (and in some cases also explicit values) for entropies of a particular class of endomorphisms of Cuntz algebras, so-called permutative endomorphisms.

Some further questions will also be indicated.

27. **Michael Skeide (Campobasso)**

What are spatial CP-semigroups?

Abstract: Following Powers, spatial E_0-semigroups $B(H)$ are semigroups of unital endomorphism that admit an intertwining semigroup of isometries in $B(H)$. This means precisely that the associated Arveson system is spatial. A third characterization is that the E_0-semigroup is cocycle conjugate to a noise. The same definition and the two characterizations (with a new generalization of the notion of cocycle conjugacy) remains true for E_0-semigroups on the algebra $B^a(E)$ of all adjointable operators on a Hilbert module E.

For CP-semigroups on $B(H)$ there are two (non-equivalent) definitions around: Arveson and Bhat define a CP-semigroup as spatial, if it dominates an elementary CP-semigroup (that is, a CP-semigroup obtained by conjugation with semigroup in the algebra), while Powers definition, still by means of intertwining semigroups of isometries, is (much!) more restrictive.

CP-semigroup that are spatial in the sense of Arveson and Bhat are those that have spatial product systems. We show by an explicit counter example that the same definition for CP-semigroups on a general C^*–algebra does not lead to necessarily spatial product systems but to product systems that may be embedded into a spatial one.

28. **Franciszek Hugon Szafraniec (Kraków)**

Naimark dilations for indeterminate moment problems.

Abstract: I intend to show going beyond von Neumann extensions (which are always available for Jacobi matrices) leads to explicit formulae for representing measures of some class of indeterminate moment problems. This is a report on joint work with Dariusz Cichoń and Jan Stochel.

29. **Piotr Śniady (Wrocław)**

Combinatorial interpretation of Kerov character polynomials.

Abstract: Free cumulants are relatively simple functionals of the shape of a Young diagram. There exist universal polynomials (called Kerov polynomials) which express the values of irreducible characters of symmetric groups in terms of free cumulants. Free cumulants are very useful for the purposes of asymptotic representation theory because the asymptotically dominant part of the Kerov polynomial has a particularly simple form, given by just one free cumulant. In my talk

I will present a simple combinatorial interpretation of the coefficients of Kerov polynomials (this result extends the work of Féray who proved the positivity of coefficients without very explicit interpretation). The main tools used in the proof are Stanley-Féray character formula and newly introduced particularly for this purpose differential calculus of functions on the set of continuous Young diagrams.

30. **Reiji Tomatsu (Tokyo & K.U.Leuven)**

On Poisson boundaries of discrete quantum groups.

Abstract: For a random walk on a discrete quantum group, Poisson boundary was abstractly introduced by Izumi. The main problem there is how a Poisson boundary is realized as a concrete non-commutative measurable space, that is, a von Neumann algebra. I will present a solution of that for an amenable discrete quantum group whose compact dual has the commutative representation ring.

31. **Janusz Wysoczański (Wrocław)**

Remarks on bm-independence.

Abstract: We introduce the general notions of bm-independence and of bm-product of algebras, and show examples. These notions are associated with partialy ordered sets of indexes, for which we study the positve symmetric cones and the Vinberg's non-symmetric cone. The classification of the symmetric cones is presented, and we study the non-commutative analogue of the classical Central Limit Theorem, for bm-independent sequences of (non-commutative) random variables. Combinatorial nature of the proofs is described. In addition, an example of an analogue of the classical Donsker's invariance principle is presented.

32. **Hiroaki Yoshida (Tokyo)**

Remarks on non-crossing "linked" partitions and free Meixner law.

Abstract: Non-crossing linked partitions can be regarded as non-crossing partitions having some links between blocks with certain restrictions, which are closely related to the multiplicative free convolutions.

In this talk, we shall give free Meixner random variables on the free Fock space, and see the higher moments can be written in terms of some statistics on non-crossing linked partitions.

We shall also see that the above construction is applicable to the q-case, that is, exactly the same form of operators on the q-Fock space will yield the q-Meixner laws and their moments also can be represented by the similar statistics on linked (not non-crossing but all) partitions.

33. **Andrzej Żuk (Paris 7)**

Growth of groups.

Abstract: We will present recent developments concerning word growth function for groups, in particular intermediate and non-uniform growth for automata groups.