ABSTRACTS OF TALKS OF 12th WORKSHOP:
NON-COMMUTATIVE HARMONIC ANALYSIS,
16 - 22.08.2009, Będlewo, Poland

1. **Michał Adamczyk (Łódź)**

   On analysis of quantum states by generalized quantum measurement.

   Abstract: Generalized quantum measurement is a fundamental obiect of noncommutative probab ility. We present new constructions of generalized measurements which allow to d istinguish all possible states of the system. The ideas are presented on elementary level of quantum systems described by comp lete operator algebras on separable Hilbert spaces.

2. **Helene Airault (Insset-LAMFA)**

   Invariant measures, unitarizing measures, the example of the Poincaré disk.

   Abstract: In the case of elementary groups, the existence of unitarizing measures is discussed.

3. **Michael Anshelevich (Texas A&M)**

   Characterizations of free Meixner distributions.

   Abstract: The free Meixner distributions are a very simple class of probability measures. Despite (or, perhaps, because of) this, they arise in a variety of contexts: random matrix theory, noncommutative limit theorems, random walks, etc. In this talk, I will indicate one reason for this, by providing a variety of *characterizations* of this class: different questions which all have the same answer, the free Meixner distributions. This will primarily be a survey of results, most of them obtained by current and past participants of this workshop.

4. **Nobuhiro Asai (Aichi)**

   Probability Measures on $\mathbb{C}$ Associated with the Jacobi-Szeg Parameters of Orthogonal Polynomials.

   Abstract: It will be discussed that probability measures on $\mathbb{C}$ associated with the Jacobi-Szeg parameters of the orthogonal polynomials can be constructed by making use of the classical Mellin transform and its convolution property. Nontrivial probability measures expressed by the modified Bessel functions will be presented as examples. Moreover, a particular choice of the index of modified Bessel functions gives us an interesting example related with the continuous dual Hahn polynomials.

5. **Teodor Banica (Toulouse)**

   The row algebra of a free quantum group.

   Abstract: It is well-known that the "row slices" of $O(n)$ are all isomorphic to the sphere S^{n-1}. An interesting problem is about what happens when $O(n)$ is replaced by a quantum group: the corresponding"noncommutative space" can be viewed/approached as a noncommutative probability space, or as an operator space, or as a spectral triple. I will describe a few such results. This is based on joint work with B. Collins and P. Zinn-Justin, and on work in progress with D. Goswami.

6. **Viacheslav Belavkin (Nottingham)**

Fock modules of Krein L–modules, affine $*$–algebras and harmonic analysis of quantum Levy–Brownian motion.

Abstract: We revisit the earlier developed algebraic approach to quantum stochastics based on the exponentiating of the Ito B^*–algebras B built over a Hilbert right C–module with the scalar product in the Lebesgue space $L = L(X)$ for a commutative $C = C(X)$. We generalize this approach replacing $L(X)$ by a predual (dual) of a noncommutative W^*–algebra (C^*–algebra) C and define the associated Krein right C–module $K + L$. We show that the Ito B^*–algebra in this case is generalized to any $*$–subalgebra B of the affine $*$–subalgebra of the right pre-Hilbert C–module K and describe the corresponding Krein–Fock module over the exponential of C. The quantum Levy–Brownian motion as the integrals $M(c, b) = \langle c, M(b)\rangle$ indexed by b from B of the integrable functions $c(x)$ is replaced by the noncommutative operator-valued integral of c in C. It is defined with values in the adjointable operators generating the exponential of the B^*–algebra B compressed onto the Fock module over K. For two basic examples of Ito–algebras B: The Heisenberg algebra of a right Hilbert L–module and the central extension $B = A + L$ of the convolutional Levy–Ito algebra A on a locally compact group G.

7. **Maciej Burnecki (Wrocław)**

Noncommutative analogs of symmetric polynomials.

Abstract: We introduce and examine several analogs of commutative symmetric polynomials, in the case of the free semigroup or the free group with a finite number of generators (Coll. Math. 1993).

8. **Marie Choda (Osaka Kyoiku)**

Fourier expansions and inner conjugacy class of subfactors.

Abstract: The free group factor $L(F_n)$ and the hyperfinite II_1 factor R are decomposed as the crossed products $N \times \mathbb{Z}_n$ of a subfactor N by the cyclic group $\mathbb{Z}_n$, where N is the fixed point algebra of an outer automorphism arising from the group. Here, we pick up such the factor inclusion $N \subset M = N \times G$. By using the Fourier expansion of a unitary $u \in M$, we give the notion of entropy $H_N(Adu)$ for inner automorphism of M with respect to N. Reminding that the inner automorphism group $\mathrm{Int}(R)$ is dense in $\mathrm{Aut}(R)$ and $\mathrm{Int}(L(F_n))$ is closed, we discuss about the inner conjugacy class $\{uNu^* : u \in M, \text{unitary}\}$ in the relation to $h(N|uNu^*)$, where $h(A|B)$ is a slight modified notion of Connes-Størmer relative entropy $H(A|B)$. This is an extended version of my talk in the Będlewo workshop (2007) that if D is the diagonal matrices then $h(D|uDu^*)$ is the entropy for the unistochastic matrices.

9. **Steve Curran (Berkeley)**

De Finetti theorems for easy quantum groups (joint work with T. Banica and R. Speicher).

Abstract: The classical groups S_n and O_n are the central objects in the study of probabilistic symmetries. De Finetti's celebrated theorem states that an infinite sequence of random variables whose joint distribution is invariant under finite permutations is conditionally independent and identically distributed. For O_n, Freedman has shown that any infinite sequence of real-valued random variables must form a conditionally independent Gaussian family with mean zero and common variance.

The free analogues S_n^+ and O_n^+ of the permutation and orthogonal groups were constructed by Wang. These are compact quantum groups in the sense of Woronowicz. Recently, C. K stler and R. Speicher discovered that de Finetti's theorem has a natural analogue in free probability: an

infinite sequence of noncommutative random variables has a joint distribution which is invariant under quantum permutations coming from S_n^+ if and only if they are freely independent and identically distributed with amalgamation, i.e. with respect to a conditional expectation. For O_n^+, we have shown that an infinite sequence of self-adjoint random variables has a joint distribution which is invariant under quantum orthogonal transformations if and only if the variables form an operator-valued free semicircular family with mean zero and common variance.

On the other hand, T. Banica and R. Speicher have introduced the class of "easy" quantum groups, with the motivating belief that "any result which holds for S_n and O_n should have a suitable extension to all easy quantum groups". In this talk we will consider 10 examples of easy quantum groups: the classical groups O_n, S_n, B_n, H_n and their free versions $O_n^+, S_n^+, B_n^+, H_n^+$, and the "half-liberated" quantum groups O_n^*, H_n^* which are neither classical or free. We will present a unified approach to de Finetti theorems for these examples, including in particular a new proof of the classical results of de Finetti and Freedman. We will also discuss a new type of independence appearing for the cases O_n^*, H_n^*, which can be modeled by certain 2×2 matrix-valued random variables with (classically) independent entries.

10. **P. K. Das (Kolkata)**

Interaction of a three–level atom with a single–mode field in a two photon resonant cavity.

Abstract: In this paper we discuss an extension of Jaynes–Cummings model by adding a further atomic level to support a second resonance and cooperative effects in multi–atom systems. A successive passage of a three–level atom in the V configuration interacting with one quantized mode of electromagnetic field in a cavity will be considered to study atomic inversion and entropy evolution of the state.

11. **Uwe Franz (Besan on)**

Characterisations of Idempotent States on Quantum Groups.

Abstract: We give several characterisations of idempotent states on finite and compact quantum groups, and classify all idempotent states on the compact quantum groups $U_q(2)$, $SU_q(2)$, and $SO_q(3)$. Furthermore we characterize all compact quantum groups whose Haar state admits a non-trivial square root. (Joint work with Adam Skalski and Reiji Tomatsu).

12. **Fumio Hiai (Tohoku)**

Riemannian metrics on positive definite matrices related to means (joint work with D. Petz).

Abstract: The Riemannian metric on the manifold of positive definite matrices is defined by a kernel function ϕ in the form $K_D^\phi(H,K) = \sum_{i,j} \phi(\lambda_i, \lambda_j)^{-1} Tr P_i H P_j K$ when $\sum_i \lambda_i P_i$ is the spectral decomposition of the foot point D and the Hermitian matrices H, K are tangent vectors. For such kernel metrics the tangent space has an orthogonal decomposition. The pull-back of a kernel metric under a mapping $D \mapsto G(D)$ is a kernel metric as well. Several Riemannian geometries of the literature are particular cases, for example, the statistical metric for multivariate Gaussian distributions and the quantum Fisher information. In this talk the case $\phi(x,y) = M(x,y)^\theta$ is mostly studied when $M(x,y)$ is a mean of the positive numbers x and y. There are results about the geodesic curves and geodesic distances. The geometric mean, the logarithmic mean and the root mean are important cases.

13. **Melanie Hinz (Wrocław)**

Multiplicative square of the free Poisson measure.

Abstract: We compute moments and free cumulants of the measure $\rho_t := \pi_t \boxtimes \pi_t$, where π_t denotes the free Poisson law with parameter $t > 0$. We also compute free cumulants of the symmetrization of ρ_t. Finally we introduce free symmetrization of a probability measure on $\mathbb{R}$ and provide some examples.

References

[1] T. Banica, S. T. Belinschi, M. Capitaine, B. Collins, Free Bessel laws. *preprint.*
[2] H. Bercovici, D. V. Voiculescu, Free convolutions of measures with unbounded support. *Indiana Univ. Math. J.* **42** (1993), 733–773.
[3] R. L. Graham, D. E. Knuth, O. Patashnik, Concrete mathematics. *Addison-Wesley* (1994).
[4] A. Nica, R. Speicher, Lectures on the Combinatorics of Free Probability. *Cambridge University Press* (2006).
[5] N. R. Rao, R. Speicher, Multiplication of free random variables and the S-transform for vanishing mean. *Elect. Comm. in Probab.* **12** (2007), 248–258.
[6] D. V. Voiculescu, K. J. Dykema, A. Nica, Free random variables. *CRM* (1992).

14. **Takeshi Hirai (Kyoto)**

Projective representations and spin characters of finite and infinite complex reflection groups.

Abstract: Let $G(m,p,n)$ be a complex reflection group and $R\big(G(m,p,n)\big)$ one of it s representation group, and let $G(m,p,\infty)$ and $R\big(G(m,p,\infty)\big)$ b e their inductive limits as $n \to \infty$. We study here projective irreducible representations (= IR) of $G(m,p,n)$ and their characters which we call *spin* characters of it. We study in particular IRs of generalized symmetric gro ups $G(m,1,n)$ and projective factor representations of $G(m,1,\infty)$ and thei r characters, which are fundamental for general $G(m,p,n)$. We also study limit ing process as $n \to \infty$. Since $R\big(G(m,1,n)\big)$ is a special central extension of $G(m,1,n)$ by $Z = H^2\big(G(m,1,n), \boldsymbol{C}^\times\big)$, projective IR π of $G(m,1,n)$ has its spin type, a character χ of Z, such that $\pi(z) = \chi(z)I$. Fixing spin types, we study in detail several cases and also the relation to t he non-spin case of $\chi = \mathbf{1}$. (Joint work with A. Hora and E. Hirai).

15. **Robin Hudson (Loughborough)**

Unitary causal and double products in non-Fock quantum stochastic calculus.

Abstract: Using non-Fock extremal universally invariant quantum stochastic calculus, unitary-valued stochastic double and causal products are characterised by their evolution and covariance properties. Canonical forms under Bogolubov transformations are found. Explicit constructions are described for each canonical form.

This is joint work with Paul Jones and Sylvia Pulmannova.

16. **Un Cig Ji (Chungbuk)**

Transformations on Boson Fock Space.

Abstract: We introduce a general transformation on Boson Fock space involving generalized Fourier–Gauss and Fourier–Mehler transforms, affine transform, Bogoliubov transformation, classical and quantum Girsanov transformations, etc. We study several intertwining properties of the transformation with the annihilation and creation operators which are applied to implementation problems for CCRs. Some interesting normal order forms and unitarity of the transformation are discussed by means of complex white noise. One–parameter transformation groups induced by the transformation and their infinitesimal generators are explicitly studied. This is based on several joint researches with N. Obata.

17. **Yasuyuki Kawahigashi (Tokyo)**

Superconformal field theory, Moonshine and operator algebras.

Abstract: Operator algebraic approach to (super) conformal field theory is presented and its relations to various topics such as subfactors, modular functions, finite simple groups, and noncommutative geometry are given.

18. **Yasuyuki Kawahigashi (Tokyo)**

Subfactors and representation theory for von Neumann algebras.

Abstract: We explain theory of bimodules over von Neumann algebras and why it is regarded as a proper representation theory for von Neumann algebras. Then we will show how it fits into theory of subfactors and quantum field theory.

19. **Dorota Kępa (Lublin)**

Statistics of animals on unbounded irregular graphs.

Abstract: We develop the idea introduced by R. L. Dobrushin in [R. L. Dobrushin: Perturbation methods of the theory of Gibbsian fields, Lecture Notes in Math. 1648, Springer, Berlin, 1996, 1-66] of controlling logarithms of partition functions in the Kotecky-Preiss animal model. It is proven that these logarithms are convergent for unbounded degree graphs with a certain property. By virtue of this property the vertices of large degree are located at large distances from each other. This property is a stronger version of the metric property introduced in [L. A. Bassalygo, R. L. Dobrushin: Uniqueness of a Gibbs Field with a Random Potential - an Elementary Approach, Theory Probab. Appl. 31, 572–589 (1987)].

20. **Louis Labuschagne (UNISA)**

Maps on noncommutative Orlicz spaces - joint work with W. A. Majewski.

Abstract: In the context of semifinite von Neumann algebras, we discuss various ways of describing noncommutative Orlicz spaces. We then generalize the construction of noncommutative Banach Function Spaces pioneered by Dodds, Dodds, de Pagter, et al, and use this generalization to a establish noncommutative version of the Pistone-Sempi argument, thereby demonstrating the utility of non-commutative Orlicz spaces for statistical physics. We then pass to the question of lifting important classes of positive maps defined on von Neumann algebras, to maps on the corresponding noncommutative Orlicz spaces. In particular, we describe those Jordan $*$-morphisms on semifinite von Neumann algebras which in a canonical way induce quantum composition operators on noncommutative Orlicz spaces. This presents strong evidence that the framework of noncommutative Orlicz spaces is well suited for an analysis of a large class of interesting noncommutative dynamical maps.

21. **Hun Hee Lee (Chungbuk)**

q-Chaos

Abstract: We will consider Khinchine type inequalities for homogeneous polynomials of q-generalized circular variables. We will first take a look at known results and will explain the main differences of our results, which contains the following. When $-1 < q < 1$ the L_p estimates for $1 \leq p \leq 2$ are essentially the same as the free case ($q = 0$), whilst the L_p estimates for $2 \leq p \leq \infty$ show a strong q-dependence. Moreover, the extremal cases $q = \pm 1$ (the CAR and CCR case) produce decisively different formulae.

22. **Franz Lehner (Graz)**

Free infinite divisibility of the normal distribution.

Abstract: We prove the somewhat strange fact that the classical normal distribution is freely infinitely divisible. This is quite unexpected as there are only two other nontrivial examples of measures with the property of being infinitely divisible both in the classical and the free sense. The proof amounts to showing that the sequence of free cumulants of the normal distribution is a moment sequence. This sequence counts the number of connected set partitions and has been studied by Touchard, Riordan and others previously. We review a few other combinatorial interpretations, from computer science to Hopf algebras of rooted binary trees which appeared in the context of renormalization theory. Despite the rich combinatorial structure a combinatorial proof still eludes us and we give a proof by analytic function theory. Joint work with S.Belinschi, M.Bozejko and R.Speicher.

23. **Eugene Lytvynov (Swansea)**

Meixner class of non-commutative generalized stochastic processes with freely independent values.

Abstract: Let T be an underlying space with a non-atomic measure σ on it (e.g. $T = \mathbb{R}^d$ and σ is the Lebesgue measure). We introduce and study a class of non-commutative generalized stochastic processes, indexed by points of T, with freely independent values. Such a process (field), $\omega = \omega(t)$, $t \in T$, is given a rigorous meaning through smearing out with test functions on T, with $\int_T \sigma(dt) f(t)\omega(t)$ being a (bounded) linear operator in a full Fock space. We define a set $\mathbf{CP}$ of all continuous polynomials of ω, and then define a non-commutative L^2-space $L^2(\tau)$ by taking the closure of $\mathbf{CP}$ in the norm $\|P\|_{L^2(\tau)} := \|P\Omega\|$, where Ω is the vacuum in the Fock space. Through procedure of orthogonalization of polynomials, we construct a unitary isomorphism between $L^2(\tau)$ and a (Fock-space-type) Hilbert space $\mathbb{F} = \mathbb{R} \oplus \bigoplus_{n=1}^{\infty} L^2(T^n, \gamma_n)$, with explicitly given measures γ_n. We identify the Meixner class as those processes for which the procedure of orthogonalization leaves the set $\mathbf{CP}$ invariant. (Note that, in the general case, the projection of a continuous monomial of order n onto the n-th chaos need not remain a continuous polynomial.) Each element of the Meixner class is characterized by two continuous functions λ and $\eta \geq 0$ on T, such that, in the $\mathbb{F}$ space, ω has representation$\omega(t) = \partial_t^\dagger + \lambda(t)\partial_t^\dagger\partial_t + \partial_t + \eta(t)\partial_t^\dagger\partial_t^2$, where $\partial_t^\dagger$ and ∂_t are the usual creation and annihilation operators at point t. This talk is based on a joint paper with Marek Bożejko.

24. **Marcin Marciniak (Gdańsk)**

On the structure of positive maps.

Abstract: The aim of the first part of the talk is to give a sort of a review of the topic. In particular we recall the general concept of a duality of ordered spaces and Stormer's and Choi's realiztions of this concept. Next we consider the notion of decomposability of positive maps and we give various conditions characterizing decomposable maps. Finaly, we discuss the facial structure of the cone of positive maps. In the second part of the talk we will use the decomposition theory (in the sense of Choquet) to give a general form of a positive map acting between type I factors. To this end we classify extremal positive maps and exposed points of the cone of positive maps from $B(K)$ into $B(H)$ in terms of some special subsets of $H \oplus K$.

25. **Alexandru Nica (Waterloo)**

Infinitesimal non-crossing cumulants and free probability of type B.

Abstract: Free probabilistic considerations of type B have been around starting with a paper by Biane, Goodman and Nica from 2003. Earlier this year Belinschi and Shlyakhtenko have connected free probability of type B to a framework called "infinitesimal free probability". My talk will present a recent joint work with Maxime Fevrier where we pursue this interplay between "type B" and "infinitesimal". Specifically, we introduce a concept of infinitesimal non-crossing cumulants (associated to an infinitesimal noncommutative probability space) which are obtained by taking a formal derivative in the formula for usual non-crossing cumulants. We prove that infinitesimal freeness of random variables is equivalent to a vanishing condition for mixed cumulants; this gives an infinitesimal counterpart for a theorem of Speicher from usual free probability. We observe how infinitesimal cumulants relate to non-crossing partitions of type B, and how they can be used to obtain infinitesimal analogues for some basic freeness results. We discuss some situations when freeness of unital subalgebras $A_1, ..., A_k$ in a noncommutative probability space (A, ϕ) can be naturally upgraded to infinitesimal freeness (e.g. in the case when one has a derivation on A which leaves every A_i invariant).

26. **Nobuaki Obata (Tohoku)**

Quantum White Noise Calculus – Quantum White Noise Derivatives and Applications.

Abstract: Quantum white noise calculus provides a systematic method for analysis of Boson Fock space operators, based on the Schwartz type distribution theory on Gaussian space. Some fundamental results on the characterization of operator symbols and Fock expansion are quickly revisited. Then, every white noise operator being considered as a function of the quantum white noise: $\Xi = \Xi(a_s, a_t^* \,;\, s, t \in T)$, we come naturally to the idea of differentiation with respect to the quantum white noise. This naive idea is formulated as the quantum white noise derivatives. Applications are found in Hitsuda-Skorohod quantum stochastic integrals, quantum Martingale representation theorem, and the implementation problem of the canonical commutation relations.

27. **Adam Paszkiewicz (Łódź)**

On notion of sufficiency in quantum statistics.

Abstract: In the general case the classical Halmos-Savage formulation of statistical sufficiency lacks the basic property: if a $\sigma-$algebra $\mathcal{A}$ is suf ficient, than also a larger $\sigma-$algebra $\mathcal{B} \supset \mathcal{A}$ i s sufficient. Some new definitions will be formulated and compared with the clas sical ones. We will consider the posibility of reconstructing a statistical spac e $\{(\Omega, \mathcal{F}, P_\theta) : \theta \in \Theta\}$ from some smaller s pace $\{(\Omega, \mathcal{A}, P_{\theta|\mathcal{A}}) : \theta \in \Theta\}$, wher e $P_{\theta|\mathcal{A}}$ is a restriction of probability measure P_θ to sub-σ-algebra $\mathcal{A} \subset \mathcal{F}$. We will also show th at our concept can be easily extended to a non-comutative case.

28. **Claudia Pinzari (Roma)**

A theory of induction for tensor C^*-categories.

Abstract: We are interested in the problem of classifying tensor C^*-categories with conjugation. If a permutation symmetry is present, a result by Doplicher and Roberts of the 90s shows that such categories are isomorphic to representation categories of compact groups. However, low dimensional QFT theories or the theory of subfactors initiated by V. Jones, provide categories with conjugation but not symmetric. Often, these can not be embedded into the category of Hilbert spaces. In the case where the object set has a distinguished generating object, we show how to associate to the given category an ergodic action of the Wang-van Daele compact quantum groups $A_o(F)$ and $A_u(F)$ on a unital noncommutative C^*–algebra. This ergodic space will be understood as a noncommutative virtual subgroup in the sense of Mackey but in a noncommutative compact

setting. We shall develop a theory of induction from this virtual subgroup which leads to an identification of the given category with a category of representations of the quantum group over Hilbert C^*–bimodules. Our results shed light on the problem of recognizing which tensor categories can be embedded into the category of Hilbert spaces. (Joint work with J.E. Roberts).

29. **Gilles Pisier (Paris 6)**

Remarks on Non-commutative Khintchine inequalities.

Abstract: We show that the validity of the non-commutative Khintchine inequality for some q with $1 < q < 2$ implies its validity (with another constant) for all $1 \leq p < q$. We prove this for the inequality involving the Rademacher functions (due to Lust-Piquard), but also for more general "lacunary" sequences, or even non-commutative analogues of the Rademacher functions. For instance, we may apply it to the "$Z(2)$-sequences" previously considered by Harcharras. The result appears to be new in that case. It implies that the space ℓ_1^n contains (as an operator space) a large subspace uniformly isomorphic (as an operator space) to $R_k + C_k$ with $k \sim n^{\frac{1}{2}}$. This naturally raises several interesting questions concerning the best possible such k. Unfortunately we cannot settle the validity of the non-commutative Khintchine inequality for $0 < p < 1$ but we can prove several would be corollaries. For instance, given an infinite scalar matrix $[x_{ij}]$, we give a necessary and sufficient condition for $[\pm x_{ij}]$ to be in the Schatten class S_p for almost all (independent) choices of signs ± 1. We also characterize the bounded Schur multipliers from S_2 to S_p. The latter two characterizations extend to $0 < p < 1$ results already known for $1 \leq p \leq 2$.

30. **Noriyosi Sakuma (Yokohama)**

$\boxplus$-Infinite Divisibility of Free Multiplicative Convolutions with Wigner and Symmmetric Arcsin Measures.

Abstract: We introduce a new subclass of probability distributions, which are constructed by $\boxtimes$ convolutions with Wigner law. We give conditions for $\boxplus$-infinite divisibilty. This talk is based on recent joint work with V. Perez-Abreu of CIMAT in Mexico.

31. **Aurel Stan (Ohio)**

Two dimensional non-commutative random vectors in terms of APC operators.

Abstract: First we introduce the joint annihilation, preservation, and creation (APC) operators of two random variables that are not commuting. We define then the notion of two dimensional non-degenerated Meixner random vectors of class ML, and classify them up to an invertible linear transformation.

32. **Franciszek Hugon Szafraniec (Kraków)**

The quantum harmonic oscillator in ℓ^2.

Abstract: Unifying the very classical processes under the umbrella of the quantum harmonic oscillator I intend to exhibit some regularities as well as peculiarities which can be viewed better when all this happens in the ℓ^2 space. This is what has been skimmed off [**?**, **?**, **?**] with some extra flavour added.

References

[s1] F.H. Szafraniec, Yet another face of the creation operator, *Operator Theory: Advances and Applications*, vol. 80, pp. 266-275, Birkh user, Basel, **1995**.

[s2] F.H. Szafraniec, Charlier polynomials and translational invariance in the quantum harmonic oscillator, *Math. Nachtr.*, **241** (2002), 163-169.

[s3] F.H. Szafraniec, Duality in the quantum harmonic oscillator, *J. Phys. A: Math. Gen.*, **34** (2001), 10487-10492.

33. **Alexander Tikhomirov (Bielefeld & Syktyvkar)**

Circular law for random matrices.

Abstract: Consider $N \times N$ random matrix with independent entries. We say that the circular law holds for some ensemble of random matrices if empirical spectral distribution convergence to the uniform distribution on the circle in complex plane as $N \to \infty$. In the Gaussian case it was known form early 60-th. Ginibre introduce ensembles of non-Hermitian matrices with i.i.d. Gaussian entries and find the density of expected joint distribution of matrix eigenvalues. From this formula Mehta has proved the circular law in complex Gaussian case (1967). For real Gaussian random variables the circular law was proved in 1995 by Edelman. In 1984 Girko and Bai in 1997 have proved this result under weak moments condition (they assume that fourth moment of entries of matrices are bounded), but they assume that distributions of entries have bounded densities. The progress in the proof of circular law for general case was arrived in the last three years. In 2007 G tze and Tikhomirov proved circular law assuming that moments of order $2 + \delta$ are bounded for some $\delta > 0$. They proved also circular law for sparse random matrices. Tao and Vu have proved the similar results. Moreover, in 2009 they proved circular law assuming finite second moment only.

34. **Reiji Tomatsu (Tokyo)**

Classification of actions of a discrete amenable Kac algebra on amenable von Neumann factors.

Abstract: In the theory of operator algebras, it is important to study (quantum) group actions, and the ultimate goal is to give a complete classification of them. We study an invariant-less action of a discrete amenable quantum group of Kac type on amenable von Neumann factors, and show its uniqueness result which generalizes the corresponding result proved by Ocneanu for a discrete amenable group.

35. **Jiun-Chau Wang (Kingston)**

New features of free central limit theorem.

Abstract: We will discuss the free central limit theorem for identical summands. Our result extends the superconvergence phenomenon to unbounded variables and even more, in addition to the uniform convergence and analiticity properties of the densities, we also obtain an entropic central limit theorem. The main tool for proving these results is the subordination functions for free additive convolution.

36. **Stanisław Woronowicz (Warszawa)**

Quantum $SL(2,\mathbb{R})$-group.

Abstract: The aim of the talk is to introduce new examples of locally compact quantum grou ps. They are deformations of classical $SL(2,\mathbb{R})$-group with the deformation parameter $q = e^{\frac{-i\pi}{4k}}$, where $k = 1, 2, \ldots$. The main steps in the c onstruction are the following:

- We start with Hopf *-algebra introduced in late eighties by Sklyagin (Leningrad school). The algebra is generated by selfadjoint elements A, B, C, D (being the matrix elements of fundamental two-dimensional representation) subject to q-commutation relations.
- We use the Zakrzewski relation to give the precise meaning to the q-comm utation relations involving selfadjoint operators.
- The Hopf algebra formulae describing comultiplication produce symmetric op erators which are not selfadjoint. We use *reflection operators* ρ_1 and ρ_2 to fix the proper selfadjoint extensions.
- We show that $\Delta A,\ \Delta B,\ \Delta C,\ \Delta D,\ \Delta\rho_1$ and $\Delta\rho_2$ satisfies the same commutation relations as A, B, C, D, ρ_1 and ρ_2.
- We introduce the C*-algebra $\mathfrak{A}$ and show that $\Delta \in \mathrm{Mor}(\mathfrak{A}, \mathfrak{A} \otimes \mathfrak{A})$.

37. **Hyun Jae Yoo (Hankyong)**

Determinantal Point Processes: Their Gibbsianness and Dynamics.

Abstract: Determinantal point processes (DPP's) are point processes on continuum or discrete state spaces whose correlation functions are given by determinants of a priori given kernel operators. We introduce some recent developments for this model. Imposing some conditions on the kernel operators we can show that DPP's are Gibbs measures for particle systems whose potential energies are given by the negative logarithms of determinants of interaction operators. Then we construct as for some equilibrium dynamics the Glauber and Kawasaki dynamics that leave certain discrete DPP's invariant.

38. **Hiroaki Yoshida (Tokyo)**

Linked partitions and deformed Meixner laws.

Abstract: Dykema introduced a new structure of partitions, the non-crossing linked partitions in his study related to the multiplicative free convolutions, which can be regarded as a non-crossing partition having some links between blocks with certain restrictions, that is, nearly disjoint. This notion of nearly disjoint can be applicable not only to the non-crossing case but to the general case, and we can obtain the linked partitions.

In this talk, we consider some deformed Fock spaces and give the deformed Meixner operators by the corresponding creation and annihilation operators on the deformed Fock space. We show the formulas for their higher moments can be expressed in terms of some set partition statistics on the linked partitions. Especially, the statistic, the number of doubly covered elements, is important for the Meixner type.

39. **Karol Życzkowski (Kraków)**

On complex Hadamard matrices and their applications.

Abstract: A real square matrix is called *Hadamard* if all its columns (and rows) are mutually orthogonal and all its entries are equal to ± 1. This notion can be generalized to give a *Complex Hadamard matrix*, which consists of unimodular entries with arbitrary phases. Complex Hadamard matrices play an important role in theoretical physics and the theory of quantum information. Straightforward construction of the Fourier matrix F_N shows that these matrices do exist for an arbitrary order N.

In a 1996 paper of Haagerup the set of Complex Hadamard matrices was completely classified for $N = 2, 3, 4, 5$. We review recent results on the structure of the set of complex Hadamard matrices of order 6 and list some related open problems.

References

[Ha96] U. Haagerup, Orthogonal maximal abelian $*$-subalgebras of the $n \times n$ matrices and cyclic n–roots, Operator Algebras and Quantum Field Theory (Rome), 1996 (Cambridge, MA: International Press) pp 296-322.

[TZ06] W. Tadej and K. Życzkowski, A concise guide to complex Hadamard matrices, Open Sys. Information Dyn. **13** (2006) 133.