ABSTRACTS OF TALKS OF 14th WORKSHOP:
NON-COMMUTATIVE HARMONIC ANALYSIS,
25.09- 1.10.2011, Będlewo, Poland

1. **Helene Airault (Université de Picardie Jules Verne, Insset & LAMFA)**

Ornstein-Uhlenbeck operators on a Kähler manifold and infinitesimal representation of a Lie group representations, some examples

Abstract: We discuss the notion of Ornstein-Uhlenbeck operator on a complex manifold endowed with a Kählerian metric. We consider the infinitesimal representation of a Lie group in a space of holomorphic functions defined on a complex domain $\mathcal{D}$ as in [1] for finite dimensional groups or as in [2] for the group of diffeomorphisms of the circle. We relate the infinitesimal representation to a Ornstein-Uhlenbeck on $\mathcal{D}$.

References:

[1] F. A. Berezin, Quantization in complex symmetric spaces, Math. USSR Izvestija, Vol. 9 (1975), n2
[2] P. Malliavin, Invariant or quasi-invariant probability measures for infinite dimensional groups, Japan. J. Math. 3, 19-47 (2008) 19-47.

2. **Hiroshi Ando (University of Copenhagen & Kyoto University)**

Embedding of Polish groups into unitary groups of II_1 factors (joint work with Yasumichi Matsuzawa at Leipzig & Hokkaido university).

Abstract: In this talk we discuss the necessary and sufficient conditions for Polish groups to be embeddable as a strongly closed subgroup into unitary groups of separable II_1 factors. Such groups are called of finite type after Sorin Popa. We also give examples of such Polish groups. This is a continuation of our study of unitary groups of finite von Neumann algebras.

3. **Octavio Arizmendi (Universität des Saarlandes)**

On a class of explicit Cauchy-Stieltjes transforms related to monotone stable and free Poisson laws

Abstract: We consider a class of probability measures $\mu^{\alpha}_{s,r}$ which have explicit Cauchy- Stieltjes transforms. This class includes a symmetric beta distribution, a free Poisson law and some beta distributions as special cases. Also, we identify $\mu^{\alpha}_{s,2}$ as a free compound Poisson law with Lévy measure a monotone α-stable law. This implies the free infinite divisibility of $\mu^{\alpha}_{s,2}$. Moreover, when symmetric or positive, $\mu^{\alpha}_{s,2}$ has a representation as the free multiplication of a free Poisson law and a monotone α-stable law. We also investigate the free infinite divisibility of $\mu^{\alpha}_{s,r}$ for r not equal to 2. Special cases include the beta distributions B(1-1/r, 1 + 1/r) which are freely infinitely divisible if and only if r is in [1,2].

4. **Nobuhiro Asai (Aichi University of Education)**

The Brenke type generating functions and orthogonal polynomials.

Abstract: In this talk, we will discuss the following classical problem:
"Determine all possible orthogonal polynomials $\{P_n(x)\}$ generated by the generating function,

$$\psi(t,x) := B(t)h(tx) = \sum_{n=0}^{\infty} h_n P_n(x) t^n, \tag{1}$$

where functions $h(x)$ and $B(t)$ are analytic around the origin,

$$h(x) = \sum_{n=0}^{\infty} h_n x^n,\ B(t) = \sum_{n=0}^{\infty} b_n t^n$$

with $h_n \neq 0$ for $n \geq 0$ and $h(0) = B(0) = 1$ just for normalizations". $\psi(t,x)$ in Eq.(1) is called the Brenke type, which is different from the Meixner type ('34). This problem was posed by Brenke ('45) and Geronimus ('47). However, they could not give a complete answer to it. Later on, Chihara ('68 '71) classified $\{P_n(x)\}$ essentially into four classes. However, explicit expressions of $\psi(t,x)$, that is, $B(t)$ and (t) for each class, were not obtained in his papers. Moreover, the associated Jacobi-Szegö parameters for Class IV were not given although several examples were mentioned briefly

by a very vague explanation. In this talk, we will report that $B(t)$ and $h(t)$ can be expressed by q-hypergeometric series and the Jacobi-Szegö parameters for Class IV coincide with those of discrete q-Hermite polynomials. The present results in this talk are based on the joint work with I.Kubo (Hiroshima, Japan) and H.-H. Kuo (LSU, USA).

5. **Michael Brannan (Queen's University, Kingston)**

Reduced von Neumann algebras of quantum permutation groups

Abstract: In 1998, Shuzhou Wang showed that the C*-algebra $C(X_N)$ of complex valued functions on the finite set $X_N = \{1, 2, \ldots, N\}$, admits a universal object in the category of quantum transformation groups acting on $C(X_N)$. The resulting universal compact quantum group is called the *quantum permutation group*, and is denoted by S_N^+.

In this talk, we will study some analytic properties of the associated reduced von Neumann algebra $L^\infty(S_N^+)$. For $N \leq 3$, S_N^+ turns out to be just the classical permutation group S_N, and therefore $L^\infty(S_N^+) \cong \mathbb{C}^{N!}$. When $N = 4$, an explicit embedding $L^\infty(S_4^+) \hookrightarrow M_4(\mathbb{C}) \otimes L^\infty(SU_2)$ is known. When $N \geq 5$, not much is known about $L^\infty(S_N^+)$, except that it is a non-injective finite von Neumann algebra. We will show that $L^\infty(S_N^+)$ always has the Haagerup approximation property. Time permitting, we will also discuss some partial results we have obtained concerning the factoriality of $L^\infty(S_N^+)$.

6. **Matthew Brown (University of Nottingham)**

The Quantum Stochastic (QS) Duhamel Principle

Abstract: We review Belavkin's theorem on the canonical pseudo-Hilbert space representation of non-commutative Itô algebras; also introduce the Guichardet-Fock space formailsm of the general QS dynamics, and the corresponding block-diagonal pseudo-Fock space representation of operator algebra in the Guichardet-Fock space. Then we introduce the point derivative operator at some present time t and establish the general form of QS derivative for non-adapted QS process - as constructed by Belavkin in the development of quantum stochastic calculus. Finally we shall consider chronological exponents as the solutions of QS differential equations (QSDEs), and present the QS Duhamel Principle for solving inhomogeneous QSDEs.

7. **Mirelle Capitaine (Institut de Mathématiques de Toulouse)**

Free subordination property and deformed matricial models

Abstract: We will show how the subordination function related to the free additive resp. multiplicative convolution allows to describe the eigenstructure of large additive resp. multiplicative spiked deformations of classical matricial models.

8. **Marie Choda (Osaka Kyoiku University)**

Entropy for unital completely posive maps

Abstract: Let $\Phi : A \to B$ be a unital completely positive (UCP) map for unital C^*-algebras A and B. We are interested in such Φ as states, conditional expectations and *-isomorphisms or automorphisms. We extend the notion of von Neumann entropy $S(\phi)$ for a state ϕ to $S(\Phi)$.

A UCP map Φ offten admits a representation $\Phi(a) = \sum_{i=1}^r v_i^* a v_i$ by means of some operators $\{v_i\}_{i=1}^r$, that is a finite operational partition of unity in the sence of Lindblad. When A, B are matrix algebras, all Φ have this form by Kraus representation or a straightforward application of Stinespring's theorem.

For each Φ of finite dimensional A and B, we show such operators $\{v_i\}_{i=1}^r$, which are unique up to $r \times r$ unitaries and r is dtermined by rank(A), rank(B) and Φ. Moreover, for another class of algebras, there are Φ given by such the $\{v_i\}_{i=1}^n$, for examples the Cuntz's canonical endmorphism on O_n and a conditional expectation of a II_1 factor onto a subfactor, etc..

Our motivation of this talk comes from the fact that entropies (due to Connes-Størmer, Connes-Narnhofer-Thirring, Brown-Voiculescu, etc.) for conditonal ecpectations or automorphisms of type I algebras are always zero. The $S(\Phi)$ is not so, and take the same value with the above entropies for a lot class of endomorphisms. Also, for an expectation Φ we have a relation between $S(\Phi)$ and Index of Φ.

9. **Joachim Cuntz (Mathematisches Institut Westfälische Wilhelms-Universität Münster)**

The regular C^*-algebras of some semigroups from number theory

Abstract: The regular C*-algebra for a semigroup has an interesting structure, in many cases comparable to that of the regular C*-algebra of a group. Important properties are in particular related to a notion of amenability. We study such algebras for the multiplicative semigroup of the ax+b-semigroup of a ring of algebraic integers. The K-theory and K-homology of these C*-algebras reflects number theoretic properties of the associated number field.

10. **José Luis da Silva (University of Madeira)**

Poisson grey noise in infinite dimensions

Abstract: In this talk we review Schneider's grey noise as well as the fractional time derivative of the survival probability function associated to the Poisson process as a motivation to introduce the Poisson grey noise measure in infinite dimensions. The Mittag-Leffer function E_α being a natural generalization of the exponential function, an infinite-dimensional version of the Poisson grey measure would have a characteristic functional

$$C_\alpha(\phi) \colon = E_\alpha\left(\int_{\mathbb{R}^d}(e^{i\phi(x)}-1)d\mu(x)\right)$$

which we prove to fulfill all requirements of the Bochner-Minlos theorem.

The identity of the support of this new measure with the support of the infinite-dimensional Poisson measure ($\alpha = 1$) allows the development of a grey infinite-dimensional analysis modeled on Poisson analysis through the combinatorial harmonic analysis on configuration spaces. This setting provides, in particular, explicit formulas for annihilation, creation, and second quantization operators. In spite of the identity of the supports, the Poisson grey noise measure displays some noticeable differences in relation to the Poisson measure, which may be physically quite significant.

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11. **Mikael de la Salle (Besançon Université de Franche-Comté)**

Non-commutative L_p spaces without the operator space approximation property.

Abstract: The hunt for examples of spaces without the approximation property in the Banach space sence goes back at least to Grothendieck and is still full of interesting open problems. But these problems are too difficult for us, we therefore study the (easier) question in the operator space setting. In the talk I will present results from a joint work with Vincent Lafforgue, where we give examples of non-commutative Lp spaces and C*-algebras without the approximation property in the operator space sense. Namely the examples are the reduced C*-algebra of SL(3,Z) and the non-commutative Lp spaces associated. The main ingredient I will present is a result of independant interest: a SO(3,R)-invariant Fourier (or Schur) multiplier for SL(3,R) automatically enjoys some regularity properties, and have a prescribed rate of decay at infinity.

12. **Maxim Derevyagin (National Academy of Science of Ukraine)**

On the λ-dynamics of the orthogonal polynomials corresponding to the Jacobi matrix $L + \lambda M$

Abstract: It turns out that some problems of combinatorics, probability theory, and mechanics lead to an eigenvalue problem of the form $(L + \lambda M - xI)q = 0$, where L, M are tridiagonal matrices and λ, x are spectral parameters. We consider the case where $L^2 = I$ and $M^2 = I$. This case is closely related to orthogonal polynomials on the unit circle and CMV matrices in the sense that every CMV matrix C admits the factorization $C = LM$. Roughly speaking, our main result is that we find the weight function corresponding to the Jacobi matrix $L + \lambda M$ explicitly. In particular, we apply the result to Jacobi polynomials on the unit circle.

13. **Takahiro Hasebe (Research Institute for Mathematical Sciences, Kyoto University)**

Semigroups related to additive and multiplicative, free and Boolean convolutions

Abstract: A one-parameter composition semigroup $\{\mathbb{B}_t\}_{t\geq 0}$ was introduced by Belinschi and Nica, defined as $\mathbb{B}_t(\mu) := (\mu^{\boxplus(t+1)})^{\uplus\frac{1}{1+t}}$. This is useful to study free infinite divisibility because a measure is freely infinitely divisible if and only if it is in the image of $\mathbb{B}_1$.

We charcterize the so-called *free divisibility indicator* $\phi(\mu) := \sup\{t > 0 : \mu \in Image(\mathbb{B}_t)\}$ in terms of Boolean convolution $\uplus$. This enables us to prove the following:

- An upper bound of $\phi(\mu)$ in terms of Jacobi parameters. This bound is achieved only by the free Meixner distributions;
- Bożejko's conjecture: if μ is freely infinitely divisible, so is $\mu^{\uplus t}$ for $0 \leq t \leq 1$;
- $\phi(\mu) = \infty$ if μ is a free or Boolean 1/2-stable law.

We introduce an analogous composition semigroup for multiplicative convolutions and define free divisibility indicators for these convolutions. Moreover, we prove that a probability measure on the unit circle is freely infinitely divisible concerning the multiplicative convolution if and only if the indicator is not less than one. We also characterize the indicator in terms of multiplicative Boolean convolution and then prove the multiplicative analogue of Bożejko's conjecture.

14. **Fumio Hiai (Tohoku University)**

Jensen and Minkowski type inequalities for operator means (joint work with J.-C. Bourin)

Abstract: Jensen inequalities for matrices and operators have various versions. The most general ones involve a unital positive linear map $\mathbb{E} : \mathbb{M}_n \to \mathbb{M}_m$. For instance, if $f(t)$ is operator concave on an interval Ω, then

$$f(\mathbb{E}(Z)) \geq \mathbb{E}(f(Z))$$

for all $Z \in \mathbb{M}_n\{\Omega\}$, the Hermitians with spectra in Ω. This is Choi's inequality, which is specialized to Hansen-Pedersen's inequality

$$f\left(\sum_{i=1}^k C_i^* Z_i C_i\right) \geq \sum_{i=1}^k C_i^* f(Z_i) C_i$$

for C^*-convex combinations in $\mathbb{M}_n\{\Omega\}$ with $n \times m$ matrices C_i such that $\sum_{i=1}^k C_i^* C_i = I$, the identity. These Jensen's inequalities are famous characterizations of operator concavity of the function f:

$$f\left(\frac{A+B}{2}\right) \geq \frac{f(A)+f(B)}{2}, \qquad A, B \in \mathbb{M}_n\{\Omega\}.$$

Are there similar inequalities by making use of the pth power map $\mathbb{E}_p(Z) := \mathbb{E}^{1/p}(Z^p)$ with $p > 0$? We first deal with this question. This contains some Jensen type inequalities for the power p-means

$$A \,\beta_p\, B := \left(\frac{A^p + B^p}{2}\right)^{1/p}$$

of two positive operators A, B.

We are also concerned with the operator means in the Kubo-Ando sense. The concavity results obtained for the means $A \,\beta_p\, B$ have analogous statements for a natural class of operator means. In particular, we obtain the Minkowski type inequality

$$\det{}^{1/n}(A \,\sigma\, B) \geq (\det{}^{1/n} A) \,\sigma\, (\det{}^{1/n} B),$$

when σ is an operator mean with some geometric convexity property. We further extends these inequalities to those involving concave functions in the general setting of symmetric anti-norms, a class of functionals on $\mathbb{M}_n^+$, including the Schatten q-anti-norms for $q \in (-\infty, 1]$ and the Minkowski functional $A \mapsto \det{}^{1/n} A$. Jensen type inequalities are obtained for symmetric anti-norms.

The operator means we treat here do not cover a wide class of Kubo-Ando means, but they turn out rather natural as they have extensions for several variables, generalizing the geometric means of several matrices introduced by Moakher and Bhatia-Holbrook. We extend some recent inequalities due to Lawson-Lim and Bhatia-Karandikar.

Finally, we discuss several basic facts on symmetric anti-norms. It is noticed that the Minkowski functional $A \mapsto \det{}^{1/n} A$ is quite a special anti-norm. We show some interpolation properties for symmetric anti-norms, with a stronger version for Schur multiplication maps. A reverse Hölder inequality involoving a symmetric anti-norm is also given.

15. **Samah Horrigue (University of Tunis El Manar)**

Quantum generalized fractional evolution equation

Abstract: In this paper, we introduced and study the generalized quantum Gross Laplacian $\Delta_{QG}(K_1, K_2)$. Then, we give an explicit solution of the quantum Riemann-Liouville (resp. Caputo) time fractional evolution equation by

applying the convolution calculus on a suitable distribution space. We show that the explicit solution is given as the convolution between the initial condition and a generalized function related to the Mittag-Leffler function.

References:

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16. **Robin Hudson (Loughborough University)**

Quantum Lévy area

Abstract: Lévy's stochastic area is that swept out by two dimensional Brownian motion, or equivalently two independent one-dimensional Brownian motions X and Y. The "momentum" and "position" Brownian motions P and Q of quantum stochastic calculus do not commute, but are nevertheless "independent" in the sense that joint characteristic functions factorize. We investigate replacing X and Y by P and Q, for example in the Lévy area formula for the characteristic function, using both Fock and non-Fock quantum Brownian motions. There are connections with Meixner distributions and random matrices.

17. **Un Cig Ji (Chungbuk National University)**

Quantum Stochastic Calculus Associated with Quadratic Quantum White Noises

Abstract: In this talk, we study the quantum stochastic calculus associated with a class of fundamental quantum stochastic processes including quadratic quantum white noises. The quantum Itô formula and unitary conditions of solutions of quantum stochastic differential equations associated with the fundamental processes are examined. The quantum stochastic calculus extends the Hudson-Parthasarathy quantum stochastic calculus and provides new stochastic dilations of quantum dynamical semigroup. This talk is based on a joint work with Professor K. B. Sinha.

18. **Alina Kargol (Maria Curie-Skłodowska University, Lublin)**

Decay of Correlations in Ferromagnetic Quantum Models

Abstract: For the ferromagnetic N-dimensional quantum model on the lattice $\mathbb{Z}^d$ ($d \geq 1$) with long-range interactions, the estimate for the two-point correlation functions $g_N(l, l', \tau, \tau')$ has been proven. It has been shown that above the critical temperature of the 1-dimensional model, the decay of $g_N(l, l', \tau, \tau')$ is the same as the decay of $J(l) = J_0/(|l - l'| + 1)^{\alpha d}$, $J_0 > 0$, $\alpha > 2$. For 'large enough' temperatures, the decay of the two-point correlation function $g_N^{\mathrm{cl}}(l, l')$ for the classical model has been obtained.

19. **Yasuyuki Kawahigashi (University of Tokyo)**

N=2 superconformal field theory and noncommutative geometry

Abstract: We describe operator algebraic approach to chiral N=2 superconformal field theory. Based on some analogy of this theory to classifical differential geometry, we present new connections to noncommutative geometry. This connects subfactors and entire cyclic cohomology in noncommutative geometry.

20. **Andrzej Komisarski (University of Łódź)**

On minimal number of measurements, which form an informationally complete system

Abstract: An elementary measurement in $\mathbb{H} = \mathbb{C}^d$ is described by a system $\mathcal{M}$ of 1-dimensional projections $\hat{f}_k = |f_k\rangle\langle f_k|$, $0 \leq k < d$, when (f_k) is an O.N. basis. A system of elementary measurements $\mathcal{M}^i = (\hat{f}^i_k)_{0\leq k<d}$, $0 \leq i < n$ is informationally complete if numbers $\left\langle f^i_k|\rho f^i_k\right\rangle_{0\leq k<d}^{0\leq i<n}$ uniquely determine the operator $\mathrm{B}(\mathbb{H}) \ni \rho \geq 0$, $\operatorname{tr}\rho = 1$. We show that informationally complete measurements exist if $n \geq d+1$. The question was posed by professor Parthasarathy and is related to so called Wootters problem.

21. **Anna Kula (Jagiellonian University, Kraków)**

Lévy Processes on compact quantum groups: from symmetries to derivation and beyond

Abstract: In the talk, we discuss the invariance and symmetry properties of generators of Lévy processes on compact quantum groups which reflect the symmetries of the process. For KMS-symmetric processes we construct a Dirichlet form, which gives rise (under further restrictions) to a derivation and a Dirac operator. We study some examples of such generators.

This is a joint work with U. Franz and F. Cipriani.

22. **Seung-Hyeok Kye (Seoul National University)**

Product vectors and their partial conjugates

Abstract: A simple tensor $x\otimes y$ in the tensor product space $\mathbb{C}^n\otimes\mathbb{C}^m$ is said to be a product vector. The partial conjugate of a product vector $x \otimes y$ is nothing but the product vector $\bar{x} \otimes y$, where $\bar{x}$ is the vector whose entries are given by the complex conjugates of the corresponding entries. The notion of product vectors and their partial conjugates play key roles in the theory of entanglement in quantum physics. We say that a pair (D, E) of subspaces in $\mathbb{C}^n \otimes \mathbb{C}^m$ satisfies the range criterion for separability if there exists a family $\{x_\iota \otimes y_\iota\}$ of product vectors such that

$$D = \text{span}\{x_\iota \otimes y_\iota\}, \qquad E = \text{span}\{\bar{x}_\iota \otimes y_\iota\}.$$

This talk consists of two parts: In the first part, we explain in what contexts the above notion arise, and examines the pairs satisfying the above criterion in the case of $\mathbb{C}^2 \otimes \mathbb{C}^n$. This will be applied to characterize the faces of the cone of all completely positive linear maps from M_n into M_2 which are still exposed faces of the much bigger cone consisting of all decomposable positive linear maps. In the second part, we consider the case when the criterion is violated in an extreme way, that is, the case when there is no product vector in D whose partial conjugate lies in E. This is an important step to classify entangled edge states with positive partial transposes by their range dimensions. When D and E be subspaces with codimensions k and ℓ, respectively, we show that if $k + \ell < m + n - 2$ then there must exist a product vector in D whose partial conjugate lies in E. If $k + \ell > m + n - 2$ then there may not exist such a product vector. If $k + \ell = m + n - 2$ then both cases may occur depending on k and ℓ. Our proof involves techniques from algebraic geometry and binomial coefficients.

23. **Christian Le Merdy (Besançon Université de Franche-Comté)**

Operators and their square functions on noncommutative L_p–spaces.

Abstract: A simple tensor $x\otimes y$ in the tensor product space $\mathbb{C}^n\otimes\mathbb{C}^m$ is said to be a product vector. The partial conjugate of a product vector $x \otimes y$ is nothing but the product vector $\bar{x} \otimes y$, where $\bar{x}$ is the vector whose entries are given by the complex conjugates of the corresponding entries. The notion of product vectors and their partial conjugates play key roles in the theory of entanglement in quantum physics. We say that a pair (D, E) of subspaces in $\mathbb{C}^n \otimes \mathbb{C}^m$ satisfies the range criterion for separability if there exists a family $\{x_\iota \otimes y_\iota\}$ of product vectors such that

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24. **Franz Lehner (Technische Universität Graz)**

Characterization Problems in Free Probability

Abstract: We present characterizations of the semicircle law by freeness of linear and quadratic forms without boundedness assumptions. Joint work with G. Chistyakov and F. Goetze.

25. **Romuald Lenczewski (Wrocław University of Technology)**

Matricial R-transform

Abstract: I will discuss the concept of the matricial R-transform which linearizes the strongly matricially free convolution. It is a linear combination of Voiculescu's R-transforms in free probability with coefficients given by internal units of the considered array of subalgebras. A similar linearization property holds for the matricially free convolution. Strong matricial freeness unifies the main types of noncommutative independence and thus the matricial R-transform plays the role of a unified noncommutative analog of the logarithm of the Fourier transform.

26. **Andrzej Łuczak (University of Łódź)**

Cloning and broadcasting states

Abstract: Let $\mathcal{A}$ be a C^*-algebra and let $\mathcal{A}\otimes_{\min}\mathcal{A}$ be the injective tensor product. A linear map $\Pi\colon \mathcal{A}^*\to(\mathcal{A}\otimes_{\min}\mathcal{A})^*$ sending states to states is said to *clone* a state ρ if $\Pi\rho=\rho\otimes\rho$. It is said to *broadcast* ρ if

$$(\Pi\rho)(a\otimes\mathbf{1})=(\Pi\rho)(\mathbf{1}\otimes a)=\rho(a),\qquad \text{for each } a\in\mathcal{A}.$$

A family Γ of states is said to be *cloneable* (resp. *broadcastable*) if there is a map Π that clones (resp. broadcasts) each member of this family. For example, the celebrated no-cloning theorem and its further generalizations say that this is impossible for Γ being the set of all states of the algebra $\mathbb{B}(\mathcal{H})$ of all linear operators on a finite dimensional Hilbert space. We want to present a number of results on cloning and broadcasting of states in the general C^*- (or W^*-) algebra framework in arbitrary dimension considerably generalizing and completing the existing ones.

27. **Eugene Lytvynov (Swansea University)**

Meixner's orthogonal polynomials in classical and non-commutative probability

Abstract: The Meixner class of orthogonal polynomials includes Hermite, Charlier, Laguerre, Meixner, and Meixner-Pollaczek polynomials. They were classically characterized by Meixner as those orthogonal polynomials whose generating function is of exponential type (and additionally the measure of orthogonality has infinite support). It appears that Meixner's polynomials, or rather their infinite dimensional counterparts, play a significant role in infinite dimensional analysis. We will start this talk by discussing how these polynomials may be characterized within the class of all orthogonal polynomials of infinitely many variables which are related to Levy processes. We will then discuss how a similar characterization theorem may be derived within the framework of free probability, leading to a free Meixner class of orthogonal polynomials of infinitely many noncommutative variables related to free Levy process. We will finish the talk with a discussion on whether similar results could be expected for q-deformations of commutation relations where $-1<q<1$ or $q\in\mathbb{C}$ with $|q|=1$, the latter being called an anyon statistics.

28. **Ion Nechita (CNRS, Université de Toulouse)**

Block-modified Wishart matrices and applications to entanglement theory

Abstract: We study linear maps acting on the blocks of a Wishart matrix with a tensor product structure. We obtain a free probabilistic description of the limiting eigenvalue distribution of the resulting matrix and we apply these results to study the notion of entanglement in quantum information theory. This is joint work with Teodor Banica, Benoit Collins and Deping Ye.

29. **Maciej Nowak (Jagiellonian University, Kraków)**

Multiplication law and S transform for non-hermitian random matrices

Abstract: We derive a multiplication law for free non-hermitian random matrices allowing for an easy reconstruction of the two-dimensional eigenvalue distribution of the product ensemble from the characteristics of the individual ensembles. We define the corresponding non-hermitian S transform being a natural generalization of the Voiculescu S transform. In addition we extend the classical hermitian S transform approach to deal with the situation when the random matrix ensemble factors have vanishing mean including the case when both of them are centered. We use planar diagrammatic techniques to derive these results.

30. **Nobuaki Obata (Tohoku University)**

Spectral analysis of Manhattan products of digraphs

Abstract: The Manhattan street network was introduced by Maxemchuk (1987) and Morillo et al. (1985) for simple and effective structure of communication networks. The spectra of the Manhattan street networks are described by Comellas et al. (2007,2008). The Manhattan product of digraphs is a natural generalization of the Manhattan street network. In this talk we review the Manhattan product of digraphs from the viewpoint of spectral analysis. We derive some preliminary formulae and show some simple examples.

31. **Janosch Ortmann (Warwick Mathematics Institute)**

Large Deviations for Non-crossing partitions and applications to free probability

Abstract: A large deviations principle for the empirical law of the block sizes of a uniformly distributed non-crossing partition is established. As an application we obtain a variational formula for the maximum of the support of a compactly supported probability measure in terms of its free cumulants, provided these are all non-negative. This is useful in free probability theory, where sometimes the R-transform is known but cannot be inverted explicitly to yield the density.

32. **Adam Paszkiewicz (University of Łódź)**

On general form of quantum information

Abstract: We develop some techniques for Boolean sublattices of $\mathrm{Proj}\,\mathbb{H}$, $\dim \mathbb{H} = \infty$, then we manage to describe a general form of information I on partitions $(P_1, \ldots, P_m)$ of $1_{\mathbb{H}}$ with mutually orthogonal projections. Such information I is any function satisfying

$$I(P_iQ_j; i \leq m, j \leq n) = I(P_i; i \leq m) + I(Q_i; i \leq n),$$

for $P_iQ_j = Q_jP_i$ and $\mathrm{tr}\,\rho P_iQ_j = \mathrm{tr}\,\rho P_i \cdot \mathrm{tr}\,\rho Q_j$, for the state ρ describing quantum system.

33. **Michael Skeide (Universita degli Studi del Molise, Campobasso)**

Free Product Systems

Abstract: Finding dilations of Markov semigroups to cocycle perturbations of noises is a fundamental problem of quantum probablity and quantum dynamics. A noise is hereby a reversible quantum dynamics with a filtration that is independent in some notion of quantum independence. The noise is a dilation of the trivial Markov semigroup that is turned into a dilation of a nontrivial Moarkov semigroup by perturbation with a unitary cocycle.

In 2004, we pointed out that, in a sense, every noise has a filtration that is conditional (or amalgamated, or operator-valued)) monotone independent; it is a monotone noise. But there may be filtration that are independent on other notions of independences, for instance, in conditional (or amalgamated) free independence.

In 2009, we proved that a Markov semigroup admits a dilation to a cocycle perturbation of a noise if (and, obviously, only if) it is spatial. A Markov semigroup is spatial if it "dominatesąn ęlementaryĆP-semigroup. Spatiality of as Markov semigroup is reflected by its GNS product system and, in fact, in the construction of the dilation the GNS-system and the classification of E_0-semigroups by their product systems up to śtablećocycle conjugacy plays a crucial role. But this construction is "gereral abstract nonsense". In particular, it is unclear if the unitrary cocycle is adapted to a suitable independent filtration or if it fulfills a quantum stochastic differential equation.

After recalling some details about these facts, we introduce free product systems an illustrate how to contruct a conditional (or amalgamated) free noise from every free productr system. We show how every spatial (tensor!) product system generates a free product system. For instance, as we pointed out in 2001, the time ordered product systems (that is, the spatial type I systems) generate the free product system of full Fock modules with the same index. For uniformly continuous continuous Markov semigroups, we constructed suitable unitary cocycles with our quantum

stochastic calculus on the full Fock module from 2000. Here, we ask if it is possible to do the same, purely algebraically without any caclulus, starting from the GNS-systsem of an arbitrary spatial Markov semigroup.

34. **Piotr Śniady (Polish Academy of Sciences, Warsaw)**

Trajectories of jeu-de-taquin (joint work with Dan Romik)

Abstract: Jeu-de-taquin is a transformation of Young tableaux which, roughly speaking, can be described as follows. For a given Young tableau we remove the corner box and let other boxes slide in the unique way. The boxes which have moved form a "trajectory of the avalanche". We consider a random infinite Young tableau. In this case the trajectory turns out to converge almost surely to a straight line with a random slope. The value of this slope can be interpreted in terms of the Robinson-Schensted-Knuth algorithm. In this way one can get new information about the dynamical system given by jeu-de-taquin, representations of the infinite symmetric groups and non-colliding random walks.

35. **Alexander Soshnikov (University of California, Davis)**

On fluctuation of matrix entries of regular functions of Wigner matrices and outliers in the spectrum of finite rank perturbations.

Abstract: I will explain the main results of three recent preprints "Fluctuations of Matrix Entries of Regular Functions of Wigner MatricesąrXiv:1103.1170 math.PR, Ón Finite Rank Deformations of Wigner MatricesąrXiv:1103.3731 math.PR, and Ón Fluctuations of Matrix Entries of Regular Functions of Wigner Matrices with Non-Identically Distributed EntriesąrXiv:1104.1663 math.PR written jointly with Sean O'Rourke, Alessandro Pizzo, and David Renfrew.

The first half of the talk will be devoted to the fluctuation of the outliers in the spectrum of finite-rank deformations of Wigner matrices. In particular, I will explain that this problem can be reduced to the question about fluctuation of the resolvent matrix entries of a standard Wigner matrix. The second part of the talk will be devoted to the fluctuations of matrix entries of regular functions of Wigner (and sample covariance) random matrices.

36. **Roland Speicher and Moritz Weber (Universität des Saarlandes)**

Quantum Groups Made Easy; featuring "the lost quantum group"

Abstract: In the first part of the talk Speicher will motivate and introduce some ęasy"quantum groups which describe symmetries in a non-commutative context. In the second part, Weber will give some ideas of the classification of those quantum groups, with special emphasis on the recently discovered łost quantum group".

37. **Fedor Sukochev (University of New South Wales, Sydney)**

On the Rosenthal inequalities in noncommutative symmetric spaces

Abstract: Probabilistic inequalities for independent random variables and martingales play a prominent role in many different areas of mathematical research, such as harmonic analysis, probability theory, Banach space geometry and the study of symmetric function spaces. In the recent years, many of these classical probabilistic inequalities have been generalized to the context of noncommutative L_p-spaces. In the context of operator algebras, these inequalities are equally fundamental for the study of the geometry of noncommutative L_p-spaces, free probability theory and noncommutative harmonic analysis. In this talk, we describe various extensions of these inequalities to the realm of noncommutative symmetric Banach function spaces. Our attention is focused on generalizing some classical inequalities for independent random variables, due to H.P. Rosenthal. The latter inequalities can be viewed as generalizations of classical Khintchine inequalities and we shall expound this connection and describe some recent developments in the area of noncommutative Khintchine inequalities as well. Joint work with S. Dirksen, D. Potapov and B. de Pagter.

38. **Fedor Sukochev (University of New South Wales, Sydney)**

On the Rosenthal-Johnson-Schechtman inequalities in symmetric function spaces

Abstract: Probabilistic inequalities for independent random variables and martingales play a prominent role in many different areas of mathematical research, such as harmonic analysis, probability theory, Banach space geometry and the study of symmetric function spaces. We present an important generalisation of the classical Khinchine inequality due to H.P. Rosenthal (1970). Recall that the Khintchine inequality holds for L_p-spaces, $1 < p < 1$ on a probability space and it asserts that any sequence of of independent Bernoulli random variables (with probability $1/2$ taking values ± 1) in such spaces is equivalent to the standard vector basis of the Hilbert space l_2. The Rosenthal inequality was the first attempt to extend this result to more general sequences of random variables than Bernoulli random variables and it

has been of profound influence in Banach space geometry. The restatement of the Rosenthal inequality due to Johnson and Schechtman (1989) is shown to be of crucial importance for further generalisations and various applications in symmetric function spaces (which may be viewed as generalisations of L_p-spaces). In this talk, we present a criterion for the validity of the Johnson-Schechtman inequalities (for general symmetric spaces) in terms of boundedness of the Kruglov operator. The latter operator was introduced by S. Astashkin and the speaker (2004) and was preceded by deep probabilistic ideas of V.M. Kruglov (1970) and subsequent applications of these ideas in Banach space geometry by M. Sh. Braverman (1994). The talk is based on the survey article Astashkin, S. V.; Sukochev, F. A. *Indepen- dent functions and the geometry of Banach spaces,* (Russian) Uspekhi Mat. Nauk 65 (2010), no. 6(396), 3–86; translation in Russian Math. Surveys 65 (2010), no. 6, 1003–1081. We refer to this survey for more information on Kruglov operator and its applications.

39. **Stanisław Woronowicz (University of Warsaw)**

Multiplicative unitary for the quantum double

Abstract: Quantum double of a quantum group G is a quantum group that combines G and the dual $\widehat{G}$ in a special way. We shall present a new formula that describe a multiplicative unitary for the double group in terms of multiplicative unitary of the original group. The formulae known till now used in an essential way the Haar weights on G and $\widehat{G}$, our formula is independent of the existence of the Haar measure.

40. **Janusz Wysoczański (University of Wrocław)**

Introduction to generalized (anyon) statistics (joint work with M. Bozejko and E. Lytvynov)

Abstract: For a hermitian kernel Q, with $|Q(x,y)| = 1$, we define the Q-deformed Fock space and related Q-creation and Q-annihilation operators. The Q-creation and Q-annihilation at points are also given meanings, and for these we obtain some deformed commutation relations. Then we study Q-Hermite and Q-Charlier orthogonal polynomials related to Q-Gaussian and Q-Poisson processes. The notions of Q-cummulants and also of Q-independence are studied.

41. **Quanhua Xu (Besançon Université de Franche-Comté)**

Harmonic analysis on noncommutative tori

Abstract: Noncommutative tori are fundamental examples in operator algebras and noncommu- tative geometry. This talk will present a systematic study of harmonic analysis on noncommutative tori. The results presented will include those on maximal inequalities, pointwise convergence of different summations, Fourier multipliers and Hardy spaces.

42. **Hiroaki Yoshida (Ochanomizu University, Tokyo)**

Integral representations of the relative free entropy associated with semicircular gradients

Abstract: Using the logarithmic energy with the potential function, the free analogue of the relative entropy between two compactly supported probability measures on the real line was introduced by Biane and Speicher. In this talk, we shall give an integral representation of the free relative entropy associated with semicircular (free gaussian) gradient flows. Furthermore, we shall introduce another integral representation related to the relative free Fisher information, which also reflects some free analogous properties of the classical relative entropy.

43. **László Zsido (Dipartimento di Matematica, Universita di Roma "Tor Vergata")**

Relative weakly mixing for non-commutative dynamical systems

Abstract: Let M be a von Neumann algebra on a Hilbert space H, $\xi_o \in H$ a cyclic and separating vector for M, and Φ a $*$-endomorphism of M leaving invariant the positive linear form $\varphi : M \ni a \longmapsto (a\,\xi_o|\,\xi_o)$. The dynamical system (Φ, φ) is called *weakly mixing* if, for every $a_o, a_1 \in M$,

$$\varphi\big(a_o\,\Phi^k(a_1)\big) \longrightarrow \varphi(a_o)\,\varphi(a_1)$$

in density (that is avoiding a natural number set of zero density). But weakly mixing can also be defined relative to any von Neumann subalgebra $N \subset M$ with $\Phi(N) = N$, which is left invariant by the modular automorphism group of φ: indeed, in this situation there is a (unique) conditional expectation $E : M \longrightarrow N$ leaving invariant φ and for the *weakly mixing relative to* N we can require the convergence

$$E\big(a_o\,\Phi^k(a_1)\big) - E(a_o)\,\Phi^k\big(E(a_1)\big) \xrightarrow{\text{so}} 0$$

in density for every $a_o, a_1 \in M$. In this talk we intend to discuss thoroughly relative weakly mixing, in particular, the existence of some N such that (Φ, φ) is weakly mixing relative to N, while the restriction of Φ to N is almost-periodic.