



**17th Workshop: Noncommutative Probability**  
**Lévy processes and operator algebras, with applications**  
**Będlewo, July 24–30, 2016**

**ABSTRACTS**

**1. Luigi Accardi (Università degli Studi di Roma "Tor Vergata")**

Some follow ups of a joint paper with Marek Bożejko

**Abstract:** I have only one joint paper with Marek Bożejko, but this paper generated several far from trivial consequences. The most important one, from the conceptual point of view, is that it radically changed our way of looking at quantum probability. I will try to explain why.

**2. Michael Anshelevich (Texas A&M University)**

The exponential homomorphism in non-commutative probability.

**Abstract:** The wrapping transformation is easily seen to intertwine convolutions of probability measures on the real line and the circle. It is also easily seen to not transform additive free convolution into the multiplicative one. However, we show that on a large class  $L$  of probability measures on the line, wrapping does transform not only the free but also Boolean and monotone convolutions into their multiplicative counterparts on the circle. This allows us to prove various identities between multiplicative convolutions by simple applications of the additive ones. The restriction of the wrapping to  $L$  has several other unexpected nice properties, for example preserving the number of atoms. This is joint work with Octavio Arizmendi.

**3. Octavio Arizmendi (Centro de Investigación en Matemáticas)**

On Distance k-Graph of Star and Free Products of Graphs

**Abstract:** In this talk we explain recent results, in collaboration with Tulio Gaxiola, on the k-distance graph of star and free products of graphs. Let  $G$  be a finite connected graph, we show that the large  $N$  limit of the spectral distribution of the distance  $k$ -graph of the  $N$ -fold star power of  $G$  converges to a centered Bernoulli distribution. Similarly, we prove that the asymptotic distributions of distance- $k$  graphs of the  $N$ -fold free product graph of  $G$ , as  $N$  tends to infinity, is given by the distribution of  $P_k(s)$ , where  $s$  is a semicircle random variable and  $P_k$  is the  $k$ -th Chebychev polynomial. We also, calculate the distribution with respect to the vacuum state of the distance- $k$  graph of a  $d$ -regular tree. From this result we are able to describe the limiting spectral distribution of the distance- $k$  graph of a  $d$ -regular random when the number of vertexes is large.

**4. Nobuhiro Asai (Aichi University of Education)**

Distribution of the field operator on the  $(\alpha, q)$ -deformed Fock space

**Abstract:** We shall give a quick review of the distribution of the field operator on the  $(\alpha, q)$ -deformed Fock space (Fock space of type B) by Bożejko-Ejsmont-Hasebe (JFA, 2015) and present recent results by Asai-Bożejko-Hasebe (JMP, 2016) on the corresponding radial Bargmann distribution.

**5. Hari Bercovici (Indiana University)**

Conjectures related to the Connes embedding problem

**Abstract:** It is known that one can associate with each pair of selfadjoint elements in  $II_1$  factor an essentially combinatorial object known as a hive. Making this association more constructive offers a possible approach to the embedding problem for such factors. We will discuss one approach to an extremal"part of the collection of hives.

## 6. Florin Boca (University of Illinois at Urbana Champaign)

## Statistics of Farey fractions and distribution of eigenvalues in large sieve matrices

**Abstract:** We will show how existing results on correlations of Farey fractions can be used to study the distribution of eigenvalues in large sieve matrices. The objects that we will discuss are elementary to define. This is joint work with Maksym Radziwill.

## 7. Włodek Bryc (University of Cincinnati)

The local structure of  $q$ -Gaussian processes

**Abstract:** The local structure of  $q$ -Ornstein-Uhlenbeck processes and  $q$ -Brownian motions are investigated, for all  $q \in (-1, 1)$ . These are the classical Markov processes corresponding to the noncommutative  $q$ -Gaussian processes. These processes have discontinuous sample paths, and the local small jumps are characterized by a tangent processes. It is shown that for all  $q \in (-1, 1)$ , the tangent processes at inner domain are scaled Cauchy processes possibly with drifts, and the tangent processes at the boundary of the domain are related to the free  $1/2$ -stable law via Biane's construction.

## 8. Marie Choda (Osaka Kyoiku University)

## Operational extreme points of completely positive maps

**Abstract:** We give a generalized version of the notion (which we call here numerical operational extreme points) in my previous paper. We discuss on these two notions, and show that a unital  $*$ -homomorphism is an operational extreme point in the operational convex set of completely positive maps on a unital  $C^*$ -algebra. A typical example is the canonical shift on the Cuntz algebra.

## 9. Vito Crismale (Università degli Studi di Bari Aldo Moro)

## Generalized ergodic properties for Yang-Baxter-Hecke quantisation

**Abstract:** We present sufficient conditions ensuring the strong ergodic property of unique mixing for  $C^*$ -dynamical systems arising from Yang-Baxter-Hecke quantisation and discuss whether they can be applied to some important cases including Monotone, Boson, Fermion and Boolean  $C^*$ -algebras in a unified version.

Moreover we show that the set of stationary stochastic processes are isomorphic to a segment in both the Monotone and Boolean situations, the Boolean processes enjoy the very strong property of unique mixing with respect to the fixed point subalgebra and the Monotone ones do not. If time permits, we will outline the case of symmetric states.

The results are jointly achieved with F. Fidaleo and Y.G. Lu.

## 10. Joachim Cuntz (Mathematisches Institut Westfälische Wilhelms-Universität Münster)

Left regular  $C^*$ -algebras for semigroups from number theory

**Abstract:** We describe the left regular  $C^*$ -algebras associated with semigroups arising from the ring of algebraic integers in a number field. This includes a determination of their  $K$ -theory as well as of their KMS-states.

## 11. Biswarup Das (Mathematics Research Unit, University of Oulu)

## From compact semitopological quantum semigroups to compact quantum groups: Ellis joint continuity theorem

**Abstract:** Ellis joint continuity theorem states that a locally compact semitopological group (meaning the multiplication is separately continuous only, no assumption about the inverse) is a topological group. Compact semitopological quantum semigroups were introduced by M. Daws in 2015, which are quantum versions of compact semitopological semigroups, which he used to study weak almost periodicity of Hopf von Neumann algebras. We will prove an Ellis joint continuity type result for compact semitopological quantum semigroups i.e. we will give a necessary and sufficient condition which will turn these objects into compact quantum groups.

This also extends the previously known result obtained by Murphy and Tuset, namely for a compact (topological) quantum semigroup satisfying weak cancellation laws is a compact quantum group.

Based on joint work with C. Mrozinski.

12. **Jan Dereziński (University of Warsaw)****On almost homogeneous Schrödinger operators**

**Abstract:** I will speak on Schrödinger operators with potential proportional to  $1/x^2$ , possibly with nontrivial boundary conditions at zero. This is a surprisingly rich topic with many applications.

13. **Antoine Derighetti (EPFL)****The extension property for the Fourier Algebra**

**Abstract:** Let  $G$  be a locally compact group,  $H$  a closed subgroup and  $u$  an element of the Fourier algebra of  $H$  (or of the Herz Figà-Talamanca of  $H$ ) with compact support. We discuss the possibility to extend  $u$  to an element of the Fourier algebra of  $G$  (or the Herz Figà-Talamanca of  $G$ ) with control of the norm and of the support of the extension.

14. **Wiktor Ejsmont (Wrocław University of Economics)****Restricted crossings of type B partitions**

**Abstract:** Using the Coxeter groups type B, we define operator type B and corresponding cumulants type B for their joint distributions, i.e. we express moments in terms of the cumulant using the number of restricted crossings and covers.

15. **Franco Fagnola (Politecnico di Milano)****Structure of norm-continuous quantum Markov semigroups**

**Abstract:** Starting from the celebrated papers of Gorini, Kossakowski, Sudarshan and Lindblad in 1976, the structure of uniformly continuous quantum Markov semigroups (QMSs) and their generators has been studied by several authors because it is quite general and several special classes deserve further investigation. In most of these researches, concern has been focused on the structure of the generator and the relationships between its algebraic properties and structural properties of the underlying open quantum system. In recent years, there has been a growing interest in the use of QMSs to model open quantum systems having subsystems which are not affected by decoherence.

In this talk we describe the structure of generators of norm-continuous quantum Markov semigroups with atomic decoherence-free subalgebra providing a natural decomposition of a Markovian quantum open system into its irreducible components and noiseless components. We also discuss new characterisations of the structure of invariant states and decoherence-free subsystems.

Reference: J. Deschamps, F. Fagnola, E. Sasso and V. Umanità, Structure of Uniformly Continuous Quantum Markov Semigroups. *Rev. Math. Phys.* **28** (2016), 1650003.

16. **Gero Fendler (Universität Wien)****Symmetry of group algebras and inverse closed subalgebras of  $B(H)$** 

**Abstract:** We shall review results on the symmetry of the group algebra of locally compact groups and consider applications to the inverse closedness of certain subalgebras of the bounded operators on  $L^2(G)$ .

An interesting example is the real “ $ax+b$ ” group, which is known to be a symmetric group. We shall discuss a specific example of a non-symmetric algebra related to this group. The talk is on joint work with Michael Leinert.

17. **Uwe Franz (Université de Franche-Comté, Besançon)****What is the Laplace operator on the free sphere?**

**Abstract:** We describe an approach for classifying invariant Markov semigroups on expected homogeneous spaces of compact quantum groups. The generators of these semigroups are natural candidates for Laplace operators on these noncommutative spaces. The examples of the free sphere is discussed in detail. Joint work with Biswarup Das and Xumin Wang.

## 18. Friedrich Götze (Bielefeld University)

**Asymptotic Approximations of Classical and Free Relative Entropy**

**Abstract:** We review recent results for the entropic convergence of sums of independent and free variables to Gaussian and

Wigner law using asymptotic expansions of relative entropies. In particular we discuss the case of Renyi-divergence distances in the classical setup and review the technical problems in the free case under a fourth moment for the free random variables.

This is joint work with S. Bobkov and G. Chistyakov.

## 19. Takahiro Hasebe (Hokkaido University)

**Free infinite divisibility for powers of random variables**

**Abstract:** In classical probability, Goldie, Steutel and Bondesson discovered some classes of infinitely divisible (ID) distributions which are closed with respect to powers of random variables.

In free probability, powers had been a difficult operation: There had been not many free ID random variables whose powers are free ID too. This talk shows several examples satisfying this property.

In particular, positive integer powers of standard semicircular elements are freely ID.

## 20. Fumio Hiai (Tohoku University)

**Reversibility of stochastic maps via quantum divergences**

**Abstract:** We first give a comprehensive review of different quantum divergences, including standard  $f$ -divergences, maximal  $f$ -divergences, sandwiched Rényi divergences, and  $\alpha$ - $\zeta$ -Rényi relative entropies. In connection with equality case in the monotonicity of those divergences, we discuss the reversibility problem for quantum stochastic maps. This is joint work with Milán Mosonyi.

## 21. Takeshi Hirai (Kyoto)

**Spin characters of finite and infinite generalized symmetric groups  $G(m, 1, n)$ ,  $4 \leq n \leq \infty$ .**

**Abstract:** For complex reflection groups  $G(m, p, n)$ ,  $p|m, n \geq 1$ , the study of their spin representations and spin characters is largely reduced to that of the generalized symmetric groups  $G(m, 1, n) \cong T^n \rtimes \mathfrak{S}_n$ ,  $T = \mathbf{Z}_m$ , called *mother groups* of the former. For  $4 \leq n < \infty$ , if  $m$  is odd, Schur multiplier  $Z = H^2(G(m, 1, n), \mathbf{C}^\times)$  is isomorphic to  $\langle z_1 \rangle = H^2(\mathfrak{S}_n, \mathbf{C}^\times) \cong \mathbf{Z}_2$ , and if  $m$  is even, it is isomorphic to  $\langle z_1, z_2, z_3 \rangle \cong \mathbf{Z}_2^3$ ,  $z_i^2 = e$ , where  $z_2$  is the non-trivial central element of the non-commutative double covering of  $T^n$ . Each irreducible spin representation  $\pi$  of  $G(m, 1, n)$  has its central character (called the spin type of  $\pi$ ),  $Z \ni z_i \rightarrow \varepsilon_i = \pm 1$  ( $i = 1$ , or  $i = 1, 2, 3$ ), as  $\pi(z_i) = \varepsilon_i I$  with the identity operator  $I$ . For each of spin types, we construct spin irreducible representations and obtain all the irreducible spin characters of finite  $G(m, 1, n)$ . Then, letting  $n \rightarrow \infty$ , we obtain all the spin characters of inductive limit groups  $G(m, 1, \infty)$ , as ‘limits’ of irreducible spin characters of  $G(m, 1, n)$ ,  $n < \infty$ .

Half of the cases has been already reported in [1] and [2]. This talk is based on a joint work with Prof. Akihito Hora.

- [1] T. Hirai, E. Hirai and A. Hora, Projective representations and spin characters of complex reflection groups  $G(m, p, n)$  and  $G(m, p, \infty)$ , I, in *MSJ Memoirs*, Vol. 29, Math. Soc. Japan, 2013, pp.49-122.
- [2] T. Hirai, A. Hora and E. Hirai, Projective representations and spin characters of complex reflection groups  $G(m, p, n)$  and  $G(m, p, \infty)$ , II, Case of generalized symmetric groups, *ibid.*, pp.123-272.

## 22. Robin Hudson (Loughborough University)

**Classical and quantum Lévy areas**

(delivered by Andrzej Łuczak)

**Abstract:** Lévy's stochastic area for planar Brownian motion has many connections in classical mathematics, eg. its moments are essentially the Euler numbers, and it is important in rough noise analysis.

I will consider quantum Lévy areas based on a one parameter family of “temperature” quantum planar Brownian motions whose two one-dimensional components are individually standard classical Brownian motions but jointly satisfy Heisenberg type commutation relations. These interpolate between the Fock case at zero temperature, where the moments are all zero [1], and the classical case at infinite temperature. The interpolating moments are described explicitly [2] in terms of Eulerian polynomials.

Alternatively, one may try to compute an analogue of the characteristic function as the expectation of a double product integral evaluated as the limit of discretisations [3]. Here the combinatorics is based on Catalan numbers and derivatives.

References:

- [1] S. Chen and R. L. Hudson, Some properties of quantum Lévy area in Fock and non-Fock quantum stochastic calculus, *Probability and Mathematical Statistics* 33, 425-434 (2013).
- [2] R. Hudson, U. Schauz and W. Yue, Moments of quantum Lévy areas using sticky shuffle Hopf algebras, ArXiv 1605.00730.
- [3] R. L. Hudson and Y. Pei, On a causal quantum stochastic double product integral related to Lévy area, ArXiv 1506.04294.

### 23. Zenon Jabłoński (Jagiellonian University, Kraków)

#### Some properties of the Cauchy Dual of a 2-isometry

**Abstract:** Let  $T$  be a bounded linear operator on a complex Hilbert space  $\mathcal{H}$ . We say that  $T$  is a 2-isometry, if  $I - 2T^*T + T^{*2}T^2 = 0$ . The Cauchy dual  $T'$  of  $T$  is given by  $T' = T(T^*T)^{-1}$ . In the talk we discuss several interesting facts about the Cauchy dual operator of a 2-isometric operators and present several classes of 2-isometries arising from weighted shifts on rooted and leafless directed trees.

The talk is based on joint work with A. Anand, S. Chavan and J. Stochel.

### 24. Un Cig Ji (Chungbuk National University)

#### Quantum Stochastic Integral Representations of Admissible White Noise Operators

**Abstract:** By using the Fock expansions of white noise operators, we discuss that operators in a certain class of admissible white noise operators are represented by a sum of quantum stochastic integrals against with the annihilation, creation and conservation processes of which the integrands are explicitly given in terms of quantum white noise derivatives. By using the classical and quantum correspondence, the stochastic integral representations, well-known as the Clark-Ocone-Haussmann formula, of admissible white noise functionals are discussed. This talk is based on a series of joint works with Nobuaki Obata.

### 25. Il Bong Jung (Kyungpook National University)

#### A remark on the invariant subspace problem

**Abstract:** Let  $\mathcal{H}$  be a separable, infinite dimensional, complex Hilbert space and let  $\mathcal{L}(\mathcal{H})$  be the algebra of all bounded, linear operators on  $\mathcal{H}$ . A subspace  $\mathcal{M}$  of  $\mathcal{H}$  is called an invariant subspace for  $T \in \mathcal{L}(\mathcal{H})$  if  $T\mathcal{M} \subset \mathcal{M}$ . Many problems in analysis are related to the classification of the invariant subspaces for bounded operators on  $\mathcal{H}$ . In particular, there has been considerable interest in the invariant subspace problem: does there exist a nontrivial invariant subspace  $\mathcal{M}$  for a given  $T \in \mathcal{L}(\mathcal{H})$ . Also the transitive algebra problem was raised by R. Kadison in 1995. As a variant of the transitive algebra problem, C. Pearcy(2005) suggested a problem (P): if there is a transitive subalgebra  $\mathbb{A}$  of  $\mathcal{L}(\mathcal{H})$  such that the closure of  $\mathbb{A}$  under the weak operator topology in  $\mathcal{L}(\mathcal{H})$  is not equal to the algebra  $\mathcal{L}(\mathcal{H})$ , is it true that operator norm and essential norm are equivalent on  $\mathbb{A}$ ? Note that if this can be solved affirmatively, then every  $S + K \in (\mathbf{S} + \mathbf{K})$  has a nontrivial invariant subspace, where  $(\mathbf{S} + \mathbf{K}) = \{S + K : S \text{ is subnormal and } K \text{ is compact in } \mathcal{L}(\mathcal{H})\}$ . In this talk we discuss some results related to the problem (P).

### 26. Vadim Kaimanovich (University of Ottawa)

#### Boundary behaviour of Thompson's group F

**Abstract:** The group F introduced by Richard Thompson in 1965 is the group of the orientation preserving piecewise linear dyadic self-homeomorphisms of the closed unit interval. Arguably, the most important open question about it is the one about its amenability as, due to the plethora of rather unusual properties of this group either answer would imply very interesting consequences. This problem has attracted a lot of attention, with an impressive number of failed attempts to prove either amenability or non-amenable of the group F.

A possible approach to proving non-amenability of a given group consists in showing that there are no non-degenerate Liouville random walks on it. Here we make the first step in this direction for Thompson's group F by showing that all reasonable finitely supported random walks on it have a non-trivial boundary behaviour.

## 27. Rafal Kobuszewski (University of Wrocław)

TBA

**Abstract:**

## 28. Seung-Hyeok Kye (Seoul National University)

**Separability of three qubit Greenberger-Horne-Zeilinger diagonal states**

**Abstract:** We show that the necessary condition of Guehne (2011) for (full) separability of three qubit GHZ diagonal states is also sufficient. We also get a simpler formula for the condition. Using this, we give a complete characterization of separability of three qubit GHZ diagonal states in terms of entries. The main tool is to use entanglement witnesses which are tri-partite version Choi matrices of positive bi-linear maps. This is co-work with Kyung Hoon Han.

## 29. Franz Lehner (Technische Universität Graz)

**Some characterization problems in free probability**

**Abstract:** We present some characterization problems for free quadratic forms and free regressions from recent work with G.Chistyakov and W.Ejsmont.

## 30. Michael Leinert (Ruprecht-Karls-Universität Heidelberg)

**Remarks on a result of Klaus Werner**

**Abstract:** In his PhD thesis Klaus Werner proved a result concerning interpolation and noncommutative integration. After his PhD he went into industry. His thesis was not published in a journal and so went mostly unnoticed. Similar results were obtained by other authors only later. We sketch the result and the historical development.

## 31. Romuald Lenczewski (Wrocław University of Technology)

**Limit moments of product matrices of Wishart type**

**Abstract:** I will describe the limit moments of certain product matrices of Wishart type, their unified realization in terms of matricially free creation and annihilation operators and their combinatorics.

## 32. Eugene Lytvynov (Swansea University)

**Gauge-invariant quasi-free states on the algebra of the anyon commutation relations**

**Abstract:** Let  $X = \mathbb{R}^2$  and let  $q \in \mathbb{C}$ ,  $|q| = 1$ . For  $x = (x^1, x^2)$  and  $y = (y^1, y^2)$  from  $X^2$ , we define a function  $Q(x, y)$  to be equal to  $q$  if  $x^1 < y^1$ , to  $\bar{q}$  if  $x^1 > y^1$ , and to  $\Re q$  if  $x^1 = y^1$ . Let  $\partial_x^+, \partial_x^-$  ( $x \in X$ ) be operator-valued distributions such that  $\partial_x^+$  is the adjoint of  $\partial_x^-$ . We say that  $\partial_x^+, \partial_x^-$  satisfy the anyon commutation relations (ACR) if  $\partial_x^+ \partial_y^+ = Q(y, x) \partial_y^+ \partial_x^+$  for  $x \neq y$  and  $\partial_x^- \partial_y^+ = \delta(x - y) + Q(x, y) \partial_y^+ \partial_x^-$  for  $(x, y) \in X^2$ . In particular, for  $q = 1$ , the ACR become the canonical commutation relations and for  $q = -1$ , the ACR become the canonical anticommutation relations. We define the ACR algebra as the algebra generated by operator-valued integrals of  $\partial_x^+, \partial_x^-$ . We will construct a class of gauge-invariant quasi-free states on the ACR algebra. Each state from this class is completely determined by a positive self-adjoint operator  $T$  on the real space  $L^2(X, dx)$  which commutes with any operator of multiplication by a bounded function  $\psi(x^1)$ . In the case  $\Re q < 0$ , the operator  $T$  additionally satisfies  $0 \leq T \leq -1/\Re q$ . Further, for  $T = \kappa^2 \mathbf{1}$  ( $\kappa > 0$ ), we will discuss the corresponding particle density  $\rho(x) := \partial_x^+ \partial_x^-$ . For  $\Re q \in (0, 1]$ , using a renormalization, we will rigorously define a vacuum state on the commutative algebra generated by operator-valued integrals of  $\rho(x)$ . This state is given by a negative binomial point process. A scaling limit of these states as  $\kappa \rightarrow \infty$  gives the gamma random measure, depending on parameter  $\Re q$ .

## 33. Krzysztof Nowak (Drexel University, Philadelphia)

**Low dimensional phase space parametrizations of reproducing formulae and orthonormal bases in space dimensions 1 and 2**

**Abstract:** The talk consists of three parts. In the first part we address the issue of reproducing formulae and wavelet type bases in space dimension 2 with 2 dimensional phase space parametrizations. We discuss recent results, which are the outcome of a joint work with Margit Pap. In the second part we move to historical perspectives. We present

the full classification of reproducing formulae coming out of restrictions of the extended (projective) metaplectic representation of  $\mathbb{R}^2 \rtimes \mathrm{SL}(2, \mathbb{R})$  to its connected Lie subgroups, obtained in collaboration with Filippo De Mari about two decades ago. In the third part, we address the context of any finite space dimension  $n$ . We introduce the extended (projective) metaplectic representation defined on the semi-direct product  $\mathbb{R}^{2n} \rtimes \mathrm{Sp}(2n, \mathbb{R})$ , and we make comments on progress done recently in this broad framework for generating reproducing formulae.

34. Nobuaki Obata (Tohoku University)

**Counting Walks in Graphs: A Quantum Probabilistic Approach**

**Abstract:** Counting walks in a graph is a basic and interesting problem. Let  $G = (V, E)$  be a locally finite graph and  $A$  the adjacency matrix. The matrix element  $(A^m)_{xy}$  counts the number of  $m$ -step walks connecting  $x$  and  $y$ . For  $x = y = o$  this number is expressible in the integral:

$$(A^m)_{oo} = \langle e_o, A^m e_o \rangle = \int_{-\infty}^{+\infty} x^m \mu(dx), \quad m \geq 0,$$

where  $\mu$  is a probability distribution on  $\mathbb{R} = (-\infty, +\infty)$ , called the *spectral distribution* of  $A$  at a vertex  $o$ .

The above relation suggests to study the number of walks from analytic or probabilistic aspect, in particular, from the viewpoint of quantum probability.

In this talk, we review the outline of spectral analysis of graphs on the basis of quantum probability, and report some recent results on graph products (Mellin, lexicographic, strong, and others). Some of the new results are based on the joint work with Hun Hee Lee (Seoul).

35. Izumi Ojima (Research Institute for Mathematical Sciences, Kyoto University)

**Local Gauge Invariance in Algebraic QFT**

**Abstract:** While the essence of local gauge invariance plays crucial roles in quantum field theory (QFT) relevant to modern physics, it has long been believed that this concept cannot be incorporated within algebraic QFT. In this talk, we show that local gauge invariance can be formulated consistently in algebraic QFT, when a categorical approach to the problem is adopted.

36. Habib Ouerdiane (University of Tunis El Manar)

**Holomorphic representation of Lie group and associated unitarizing measures**

**Abstract:** Consider a Lie group with a unitary representation into a space of holomorphic functions defined on a domain  $\mathcal{D}$  of  $\mathbb{C}$  and in  $L^2(\mu)$ , the measure  $\mu$  being the **unitarizing** measure of the representation.

On finite dimensional examples, we show that this unitarizing measure is also the **invariant measure** for some differential operators on  $\mathcal{D}$ .

In this talk we calculate these operators in the following elementary cases:

- A) The commutative groups  $(\mathbb{R}, +)$  and  $(\mathbb{R}^* = \mathbb{R} - 0, \times)$ .
- B) The three dimensional Heisenberg group.
- C) The affine group on the real line.

37. Adam Paszkiewicz (University of Łódź)

**On divergent products of contractions – simple ideas in complicated constructions**

**Abstract:** For some special systems of contractions  $\{Q_1, \dots, Q_k\}$  in some Hilbert (or Banach) space  $H$ , it is possible to obtain a diverging in norm sequence  $P_n \dots P_1 x$ ,  $n \in \mathbb{N}$ , for  $P_1, P_2, \dots \in \{Q_1, \dots, Q_k\}$ ,  $x \in H$ . The constructions of  $\{Q_1, \dots, Q_k\}$  was obtained by Hundal, Paszkiewicz, Kopecka-Müller, Kopecka-Paszkiewicz and Komisarski (with important additional assumptions). The examples usually was complicated. We explain some simple, basic ideas, in particular connected with dilations methods.

38. Aljosa Peperko (University of Ljubljana)

**Inequalities on the spectral radius, operator norm and numerical radius of Hadamard weighted geometric mean of positive kernel operators**

**Abstract:** Recently, several authors (K.M.R. Audenaert (2010), R.A. Horn and F. Zhang (2010), Z. Huang (2011), A.R. Schep (2011), A. Peperko (2012), D. Chen and Y. Zhang (2015), R.Drnovšek and A. Peperko(2016)) have proved inequalities on the spectral radius  $\rho$ , operator norm  $\|\cdot\|$  and numerical radius of Hadamard products and ordinary

products of non-negative matrices that define operators on sequence spaces, or of Hadamard geometric mean and ordinary products of positive kernel operators on Banach function spaces. We present generalizations and refinements of several of these results.

39. **Massimo Picardello (Universita degli Studi di Roma "Tor Vergata")**

**Universal properties of harmonic functions on trees**

**Abstract:** We consider an infinite locally finite tree  $T$  with the property that all vertices except at most a finite number have degrees at least 3, equipped with nearest neighbor transition coefficients, giving rise to a space of harmonic functions. We show that, except for trivial cases, the generic harmonic function on  $T$  has dense range in  $\mathbb{C}$ . By looking at forward-only transition coefficients, we show that the generic harmonic function induces a boundary martingale that approximates in probability all measurable functions on the boundary of  $T$ . We also study algebraic genericity, spaceability and frequent universality of these phenomena

40. **Gilles Pisier (Texas A&M University)**

**Sidon sets in bounded orthonormal systems**

**Abstract:** We will recall some of the classical theory of Sidon sets of characters on compact groups (Abelian or not). We will then give several recent extensions to Sidon sets and randomly Sidon sets in bounded orthonormal systems, following recent work by Bourgain and Lewko, and by the author, both currently available on arxiv.

41. **Marek Ptak (University of Agriculture in Kraków)**

**Asymmetric truncated Toeplitz operators and their symbols (by Marek Ptak and Kamila Kliś-Garlicka)**

**Abstract:** Let  $H^2$  be the Hardy space on the unit disc, identified as usual with a subspace of  $L^2$  on the unit circle. With any nonconstant inner function  $\theta$  we associate the model space  $K_\theta^2$ , defined by  $K_\theta^2 = H^2 \ominus \theta H^2$ .

Let us consider two nonconstant inner functions  $\alpha$  and  $\theta$  such that  $\alpha$  divides  $\theta$ . For a given function  $\varphi \in L^2$  we can define an asymmetric truncated Toeplitz operator  $A_\varphi : K_\theta^2 \rightarrow K_\alpha^2$  by  $A_\varphi f = P_\alpha(\varphi f)$ , where  $P_\alpha : L^2 \rightarrow K_\alpha^2$  is the orthogonal projection. Characterizations of bounded asymmetric truncated Toeplitz operators with  $L^2$  symbols are given in terms of rank two operators. The relations between this characterizations and the symbol of the operator will be presented.

Joint work with C. Câmara, J. Blicharz.

42. **Gerhard Racher (Universität Salzburg)**

**On invariant operators**

**Abstract:** After reviewing the existence of translation invariant operators from  $L^\infty(G)$  into some Banach  $G$ -modules we show that a locally compact group  $G$  is discrete if (and only if) there exists a non-zero  $A(G)$ -invariant operator from the left von Neumann algebra of  $G$  into its Fourier algebra  $A(G)$ .

43. **Noriyoshi Sakuma (Aichi University of Education)**

**Unimodality and selfdecomposability in classical and free probability**

**Abstract:** I will explain recent developments of study on unimodality and selfdecomposability of free infinitely divisible distributions. Unimodality of free Levy processes are similar to classical case in terms of Levy measure. But about long time behavior, we can find difference between free and classical case.

44. **Kalyan Sinha (Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore)**

**Stop-Time in Fock Space and Markov CCR- Flows**

**Abstract:** The CCR-flow gives a simple example of a semigroup of endomorphisms of the set of bounded operators in a Fock space and using a stop-time in Fock space, one can construct a large class of new flows. A useful source of such stop-times in the Classical theory is the class of additive Brownian Functionals and that in quantum probability is replaced by "Markov shifts", which allow construction of yet another family of semigroups of stopped CCR-flows.

## 45. Adam Skalski (Polish Academy of Sciences &amp; University of Warsaw)

**An application of conditionally free product of states to approximation properties of discrete quantum groups.**

**Abstract:** The conditionally free product of states, introduced by Marek Bożejko and Roland Speicher has clear roots in the study of ‘free’ product of positive definite functions, due to Marek. I will explain how one sees the same construction appear in the context of discrete quantum groups and show how we used it with M.Daws, P.Fima and S.White to prove that the free product of discrete quantum groups with the Haagerup property still has the Haagerup property.

## 46. Michael Skeide (Università degli Studi del Molise, Campobasso)

**Freeness vs Monotone Independence; from Spatial Tensor Product Systems to Free Product Systems**

**Abstract:** We focus on the interrelation between free independence (*freeness*) and monotone independence. While freeness is probably the most fundamental among all quantum independences, monotone independence has the advantage that it is always there in a quite large class of irreversible quantum dynamics, so-called *spatial* dynamics. (One might refer to monotone independence as spatial, because spatiality is so archetypal for monotone independence in dynamics.) It is, therefore, a good thing to know how to lift spatial dynamics to free dynamics. Like spatial dynamics is characterized by spatial tensor product systems, the free dynamics we obtain is characterized by free product systems.

## 47. Roland Speicher (Universität des Saarlandes)

**On deformations and independences**

**Abstract:** I will survey some work on deformations and notions of independences. Some are old, some are new; most of them are related to Marek, in one way or another.

## 48. Aurel Stan (Ohio State University)

**Some inequalities for norms of Wick products**

**Abstract:** The Wick products are, at least in the Gaussian case, intimately related to the definition of stochastic integrals. Thus, it may be important to understand inequalities about the norms of Wick products. The Wick products are also related to the quantum operators, specially the creation ones. In this talk we present some inequalities for norms of Wick products generated by the Gaussian, Poisson, and Gamma distributed random variables.

## 49. Jan Stochel (Jagiellonian University, Kraków)

**Exotic composition operators over directed graphs with one circuit.**

**Abstract:** Subnormality of composition operators in  $L^2$ -spaces over (possibly simplest) discrete measure spaces is discussed. We restrict ourselves to composition operators over connected directed graphs whose vertices, all but one, have valency one. This includes the class of weighted shifts on directed trees with one branching vertex and with infinite trunk, as well as the class of composition operators over the directed graph with one branching vertex, a circuit of length  $\kappa + 1$  and  $\eta$  branches. The former class has been intensively studied since 2012. The latter class is new. It has unexpected properties. The question of the existence of a nonhyponormal composition operator in an  $L^2$ -space over a directed graph with one loop, which generates Stieltjes moment sequences will be discussed.

## 50. Franciszek Hugon Szafraniec (Jagiellonian University, Kraków)

**The anatomy of coherent states**

**Abstract:** Following the history of the topic I intend to demystify it gradually taking advantages of Horzela- Szafraniec approach and even try to evolve it a bit beyond that.

References:

- [1] A. Horzela and F.H. Szafraniec, A measure free approach to coherent states, *J. Phys. A: Math. Theor.* **45** (2012) 244018
- [2] A. Horzela and F.H. Szafraniec, A measure free approach to coherent states refined, in Proceedings of the XXIX International Colloquium on Group-Theoretical Methods in Physics 2012, Tianjin, China, Nankai Series in Pure, Applied Mathematics and Theoretical Physics 11, 277.

## 51. Wojciech Tarnowski (Jagiellonian University, Kraków)

**Haagerup-Larsen Theorem for eigenvectors of nonhermitian random matrices - joint work with Serban Belinschi, Maciej A. Nowak, Roland Speicher**

**Abstract:** Biunitarily invariant matrices belong to the ensembles, the pdf of which is given by  $P(X) \sim \exp(-N \text{Tr} V(X X^\dagger))$ . In the  $N \rightarrow \infty$  limit these matrices become R-diagonal operators. They are invariant under the transformations  $X \rightarrow UXV^\dagger$  with  $U, V$  unitary. Such transformations bring them to the diagonal form with singular values on the diagonal. Among the symmetries of the ensemble, only the unitary transformations  $X \rightarrow UXU^\dagger$  give eigenvalues, yielding also additional parameters of the Schur decomposition. Since the transformations to the Schur form and the singular values belong to the symmetry classes, one expects a direct relation between the eigenvalues and singular values. Indeed, the mapping between mean spectral density of eigenvalues and singular values is given by the Haagerup-Larsen theorem [1]. But what about other unitarily invariant variables?

Recently [2], two of the authors gave a mathematical rigour to the quaternionization method [3,4], for non-Hermitian ensembles, broadly used in physics literature. Exploiting this results, we extend the Haagerup-Larsen theorem to describe not only the eigenvalues but also a certain correlator between left and right eigenvectors of R-diagonal operators. Such object is invariant under the unitary transformations and our results give an insight into the unexplored properties of the eigenvectors of non-normal operators.

- [1] U. Haagerup, F. Larsen, *J. Funct. An.* **176**, 331-367 (2000).
- [2] S. Belinschi, R. Speicher, P. Śniady, *Eigenvalues of non-hermitian random matrices and Brown measure of non-normal operators: hermitian reduction and linearization method*, arXiv:1506.02017 [math.OA].
- [3] R. A. Janik, M. A. Nowak, G. Papp, I. Zahed, *Nucl. Phys. B* **501**, 603-642 (1997).
- [4] A. Jarosz, M. A. Nowak, *J Phys. A* **39**, 32 (2006)

## 52. Yuri Tomilov (IMPAN)

**Why do circles in the spectrum matter?**

**Abstract:** We present several results linking the joint numerical ranges of Hilbert space operator tuples to the circle structure of the spectrum of tuples. Our approach allows us to unify and/or essentially extend several results where the circular structure of the spectrum is crucial: Arveson's theorem on almost-wandering vectors of unitary actions, Brown-Chevreau-Pearcy's theorem on invariant subspaces of Hilbert space contractions and Hamdan's recent result on supports of Rajchman measures, to mention a few.

This is joint work with V. Müller (Prague).

## 53. Anatolij Vershik (Steklov Institute of Mathematics, St. Petersburg)

**Filtration in the theory of AF-algebras and in Probability Theory**

**Abstract:** We emphasize the importance of the notion of filtration – decreasing sequence of the subalgebras in  $C^*$ -algebras and parallel notion of filtrations of sigma-fields in probabilistic and measure theoretical situations. The main class of filtrations is so called standard filtrations and corresponding class of AF-algebras. Filtration must be considered in some sense as "coapproximation" with respect to ordinary approximation theory in many areas.

## 54. Dan Voiculescu (University of California, Berkeley)

**The commutant mod a normed ideal of an  $n$ -tuple of operators**

**Abstract:** The talk will be about operator algebras which are commutants mod normed ideals of  $n$ -tuples of operators. Connections with perturbation theory, entropy and K-theory aspects will be emphasized.

## 55. Wilhelm von Waldenfels (Heidelberg)

**Results in Spectral Schwartz Distributions**

**Abstract:** A spectral Schwartz distribution is but a factor the complex conjugate Schwartz derivative of the resolvent of an operator. It solves the eigenvalue problem. It is carried by the singular points of the resolvent and associates to any such point the corresponding eigen-operator. If e.g. the operator is self-adjoint the distribution is carried by the real line and is the derivative of the usual spectral decomposition. In some important cases it can be explicitly calculated, even when the operator is not normal.

56. **Jiun-Chau Wang (University of Saskatchewan)****Limit distributions in bi-free probability theory**

**Abstract:** We will give a report on the recent progress of bi-free harmonic analysis for probability measures on the plane. The results are used to prove limit theorems for bi-free convolution. This is joint with Hao-Wei Huang and Takahiro Hasebe.

57. **Michał Wojtylak (Jagiellonian University, Kraków)****On random perturbations of matrices and matrix polynomials (joint work with P. Pagacz, Jagiellonian University)**

**Abstract:** We study the following problem. Let  $C(z) \in \mathbb{C}^{n,n}$  be a regular matrix polynomial with a given spectrum and let  $W(z) \in \mathbb{C}^{N,N}$  be a random polynomial with known spectrum with  $N$  large. What is the spectrum of

$$A(z) = PC(z)Q + W(z),$$

where  $PC(z)Q$  is an embedding of  $\mathbb{C}^{n,n}[z]$  into  $\mathbb{C}^{N,N}[z]$ ? We give some partial answers and also we estimate the resolvent  $A(z)^{-1}$ . Examples of the form:

- $z^2 A_0 + zW + A_1$ , where  $W$  is a Wigner or a Marchenko-Pastur matrix,
- $A$  is partially random port-Hamiltonian matrix,
- $A$  is random H-selfadjoint,

will be presented.

58. **Hiroaki Yoshida (Ochanomizu University, Tokyo)****Remarks on the fluctuation moments of Wigner matrices**

**Abstract:** The ensembles of Wigner matrices have the same first order limit, the semi-circle law, which can be shown by tridiagonalization. The general tridiagonal random matrix models with independent entries have been investigated by Dumitriu-Edelman and Popescu. They have given the formulas on the fluctuations off tridiagonal random matrices. Although the tridiagonalization of a Wigner ensemble, of course, yields a tridiagonal random matrix, the entries of the resulting matrix are not independent any longer. Thus we cannot apply the fluctuation formulas for the tridiagonal models to the Wigner matrices. In this talk, we shall see the second order free cumulants of Wigner matrices and derive the explicit formulas of their fluctuation moments. The combinatorial interpretation is also given in terms of the non-crossing annular pairings.

59. **Joachim Zacharias (University of Glasgow)****On embeddings of exact  $C^*$ -algebras into nuclear ones**

**Abstract:** Kirchberg showed that a separable exact  $C^*$ -algebra is isomorphic to a subquotient of the CAR algebra. Subsequently he showed that it can be embedded into a nuclear  $C^*$ -algebra, more specifically the Cuntz algebra  $O_2$  by using a lot of machinery. We'll discuss and explore the proofs. By modifying the proof of Kirchberg's subquotient result we show that every separable exact  $C^*$ -algebra embeds into the quotient of the Roe algebra over the positive integers  $\mathbb{N}$  modulo the compacts, which is a nuclear  $C^*$ -algebra thus giving a shorter proof that separable exact  $C^*$ -algebras embed into nuclear ones.