# ISOMETRY GROUPS OF INFINITELY ITERATED WREATH PRODUCTS OF METRIC SPACES

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In articles [1], [2] F.Harrary and G.Sabidussi introduced a construction of the composition of graphs. Later this construction was called the wreath product of graphs.

A notion of the wreath product of metric spaces was introduced in [4] analogously to the Sabidussi's and Harrary's one.

Recall, that metric spaces $(X, d_X)$ and $(Y, d_Y)$ are called isomorphic ([3]) if there exists a scale, that is a strictly increasing continuous function $s: \mathbb{R}^+ \to \mathbb{R}^+$, $s(0) = 0$, such that $d_X = s(d_Y)$.

Assume that there exists a positive number $r$, such that for arbitrary points $x_1, x_2 \in X$, $x_1 \neq x_2$, the inequality $d_x(x_1, x_2) \geq r$ holds. Additionally assume that the diameter $diamY$ of the space $(Y, d_Y)$ is finite. Then fix a scale $s(x)$ such that

$$diam(s(Y)) < r. \tag{1}$$

Define a metric on the cartesian product $X \times Y$ by the rule:

$$\rho_s((x_1, y_1), (x_2, y_2)) = \begin{cases} d_X(x_1, x_2), & \text{if } x_1 \neq x_2 \\ s(d_Y(y_1, y_2)), & \text{if } x_1 = x_2 \end{cases}. \tag{2}$$

The space $(X \times Y, \rho_s)$ is called the wreath product of metric spaces $X$ and $Y$ with scale $s$.

We intend to discuss some properties of the inductive and the projective limits of finitely iterated wreath products of metric spaces. These constructions are considered as infinitely iterated wreath products of metric spaces. We describe the isometry groups and some properties of infinitely iterated wreath products of metric spaces.

## References


[1] F.Harrary, *On the group of the composition of two graphs*, Duke Math J. 26 (1959), 47–51.

[2] G.Sabidussi, *The composition of graphs*, Duke Math J. 26 (1959), 693–696.

[3] I.J.Shoenberg, *Metric spaces and completely monotone functions*, The Annals of Mathematics, 39, (4) (1938), 811-841.

[4] B.Oliynyk, *Isometry groups of wreath products of metric spaces*, Algebra and Discrete Mathematics, 4 (2007), 123–130.