Nilpotent groups with poor lattice of verbal subgroups

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The talk concerns the problem of characterization of nilpotent groups of some type having poor verbal structure. A residually nilpotent group G is called verbally poor, if its every verbal subgroup coincides with a term of the lower central series in G. Then the lattice of its verbal subgroups is isomorphic to a chain L_n, i.e. the set of natural numbers of cardinality n ($n \in \mathbf{N} \cup \{\infty\}$) with a natural order.

We prove the following

Theorem 1 *The group $UT_n(K)$ of unitriangular matrices of size $n \times n$ over an arbitrary field K is verbally poor and the lattice of its verbal subgroups $\mathcal{L}at_{verb}(UT_n(K))$ is isomorphic to the chain L_{n+1}.*

We also find the width of the terms of the lower central series (as verbal subgroups) of these groups.

Then we provide necessary conditions for a finitely generated nilpotent group of nilpotency class c to have the lattice of verbal subgroups isomorphic to L_{c+1}, which are formulated in the following

Theorem 2 *A verbally poor finitely generated nilpotent group is a finite p-group with the lower central series being a p-central series.*

References

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[3] Bier, A., "On verbal subgroups of finitely generated nilpotent groups", A&DM vol. 2 (2009)