J. H. C. Whitehead's Asphericity Question

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Having enjoyed a steady diet of geometric group theory for many years, in this brief talk I would like to re-introduce a classical problem of group theory and topology that deserves to be considered by a new generation. (There are others.) In 1941, J. H. C. Whitehead asked whether asphericity is a hereditary property for two-dimensional CW complexes. The problem is still unsolved and I would not be surprised if it outlasts us all. Immediately upon posing the question, Whitehead reduced it to a question in combinatorial group theory. This algebraic question turns out to be much more difficult than the heredity question, so I will not speak about that. Instead, I will highlight some major themes in group theory and topology that directly relate to Whitehead's problem. More such themes will emerge in the future, probably in other areas and possibly soon. I will point out open questions as appropriate. Finally, as time allows I will try to illustrate why the problem is hard.