Diestel-Leader graphs and lamplighter groups

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Abstract

We introduce the Diestel-Leader graphs $DL(m, n)$, both as a limit of a sequence of graphs, and as a subgraph of the product of two trees $\mathbb{T}_{m+1} \times \mathbb{T}_{n+1}$. We show that $DL(n, n)$ is vertex transitive and can be viewed as the Cayley graph of the lamplighter group $\mathbb{Z}_n \wr \mathbb{Z}$ with the standard set of generators. We discuss some aspects of a proof by Eskin, Fisher and Whyte which asserts that:

1. If $m \neq n$, then $DL(m, n)$ is not quasi-isometric to any Cayley graph.
2. The quasi-isometry class of a Diestel-Leader graph $DL(m, n)$ is (explicitely) determined by m and n.

As a consequence, the lamplighter groups $\mathbb{Z}_n \wr \mathbb{Z}$ can be fully classified up to quasi-isometry.

The following are useful references:

1. R. Diestel, I. Leader, A conjecture concerning a limit of non-Cayley graphs. *J. Algebraic Combin.* **14** (2001) 17-25.
2. W. Woess, Lamplighters, Diestel-Leader graphs, random walks, and harmonic functions, *Combinatorics, Probability and Computing* **14** (2005) 415-433.
3. A. Eskin, D. Fisher, K. Whyte, Quasi-isometries and rigidity of solvable groups. *Pure Appl. Math. Q.* **3** (2007) 927-947