

CLIFFORD-KLEIN FORMS OF RANK-ONE SEMISIMPLE GROUPS

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As part of the general program of determining all compact quotients of homogeneous spaces G'/G , I will consider the following case. Let G be a real linear connected semisimple Lie group of rank one or the set of \mathbf{k} -points of a connected semisimple algebraic group of \mathbf{k} -rank one over a local field \mathbf{k} (*i.e.* $\mathbf{k} = \mathbb{R}, \mathbb{C}$, a p -adic field, or the field $\mathbb{F}((t))$ of formal Laurent series with coefficients in a finite field \mathbb{F}). I will consider the homogeneous space $(G \times G)/G$, where G embeds diagonally in $G \times G$; of course this homogeneous space identifies with G endowed with the action of $G \times G$ by left and right translation.

The case of $G = \mathrm{PSL}_2(\mathbb{R})$ has been extensively examined by Goldman, Kobayashi, Kulkarni and Raymond, and Salein, notably because the three-dimensional compact anti-de Sitter manifolds are the compact quotients of $(\mathrm{PSL}_2(\mathbb{R}) \times \mathrm{PSL}_2(\mathbb{R}))/\mathrm{PSL}_2(\mathbb{R})$, up to a finite covering. The case of $G = \mathrm{SL}_2(\mathbb{C})$ has also been considered by Ghys in relation to the deformations of complex structures on compact manifolds that are homogeneous under $\mathrm{SL}_2(\mathbb{C})$.

I will focus on the case when \mathbf{k} is nonarchimedean, for instance when $G = \mathrm{SL}_2(\mathbb{Q}_p)$. I will describe all torsion-free discrete subgroups Γ of $G \times G$ acting properly discontinuously on $(G \times G)/G$, and I will give a necessary and sufficient condition on Γ for the quotient to be compact. Assuming that this condition is satisfied, I will prove that after a small deformation in G' the group Γ keeps acting freely, properly discontinuously, and cocompactly on G'/G . This follows from a general result on isometric actions of free groups on simplicial \mathbb{R} -trees; one particular case of this result leads to a proof of the existence of a certain “asymetric distance” on Outer space.

References: - F. Kassel, *Proper actions on corank-one reductive homogeneous spaces*, J. Lie Theory 18 (2008), p. 961–978.
- F. Kassel, *Quotients compacts des groupes ultramétriques de rang un*, arXiv 0904.4657.