Finite groups acting on low-dimensional manifolds.

We discuss the problem of classifying finite groups which admit an action on low-dimensional manifolds satisfying certain properties in homology. In this talk we present some results about finite groups acting on homology 4-spheres (i.e. 4-manifolds with the same integer homology groups of the 4-sphere) and acyclic 5-manifolds (i.e 5-manifolds with trivial reduced integer homology); our actions are faithful, smooth and orientation preserving.

In the case of homology 4-spheres, we consider as reference model the finite subgroups of the orthogonal group SO(5), which acts on the 4-sphere S^4. We prove that the only nonabelian simple groups admitting an action on a homology 4-sphere are the alternating groups $\mathbb{A}_5$ and $\mathbb{A}_6$; both of them are isomorphic to a subgroup of SO(5). We deduce from this result a list of groups which contains all finite nonsolvable groups admitting an action on a homology 4-sphere; all the groups in the list have a subgroup of index at most two isomorphic to a subgroup of SO(5). At the moment we do not know any group acting on a homology 4-sphere which is not isomorphic to a subgroup of SO(5).

In the acyclic case the reference model is the same, in fact the standard examples are given by the action of SO(5) on $\mathbb{R}^5$. In dimension three and four we prove that a finite group acting on an acyclic manifold is isomorphic to a finite subgroup of the reference model (finite subgroups of SO(3) and SO(4) respectively). In the five dimensional case this remains true for semisimple groups. In particular a nonabelian simple group acting on an acyclic 5-manifold is isomorphic to $\mathbb{A}_5$ or to $\mathbb{A}_6$. We deduce a list of finite groups, closely related to the finite subgroups of SO(5), which are candidates for action on acyclic 5-manifolds. We note that for acyclic 5-manifolds we consider also the solvable groups. Also in this case we do not know any group acting smoothly on an acyclic 5-manifold which is not isomorphic to a subgroup of SO(5); for continuous action such examples exist, in fact some of the Milnor groups act on $\mathbb{R}^5$ but none of them is isomorphic to a subgroup of SO(5).

The talk is based on the following papers:

- M.Mecchia, B.Zimmermann, *On finite simple and nonsolvable groups acting on homology 4-spheres.* Top. Appl. 2006, 2933-2942
- M.Mecchia, B.Zimmermann, *On finite groups acting on acyclic low-dimensional manifolds.* arXiv:0808.0999v2 [math.GT]