# No-splitting property and boundaries of random groups

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The density model for random groups was introduced by Gromov. We adopt the following language from a survey by Ollivier.

**Definition.** Let $F_n$ be the free group on $n \geq 2$ generators $s_1, \ldots, s_n$. For any integer $L$ let $R_L \subset F_n$ be the set of reduced words of length $L$ in these generators.

Let $0 < d < 1$. A *random set of relators at density* $d$*, at length* $L$ is a $\lfloor (2n-1)^{dL} \rfloor$– tuple of elements of $R_L$, randomly picked among all elements of $R_L$.

A *random group at density* $d$*, at length* $L$ is the group $G$ presented by $\langle S | R \rangle$, where $S = \{s_1, \ldots, s_n\}$ and $R$ is a random set of relators at density $d$, at length $L$.

We say that a property of $R$, or of $G$, occurs *with overwhelming probability* (shortly, *w.o.p.*) *at density* $d$ if its probability of occurrence tends to 1 as $L \to \infty$, for fixed $d$.

Gromov proved that a random group at density less than $\frac{1}{2}$ is with overwhelming probability word-hyperbolic, with aspherical presentation complex. Consequently w.o.p. at density less than $\frac{1}{2}$ a random group is torsion free, of cohomological dimension 2, and its Euler characteristic is positive.

We address the following question. At density less than $\frac{1}{2}$, what is the boundary at infinity of a random group $G$?

Since $G$ is 2–dimensional, its boundary has topological dimension 1 (by the work of Bestvina and Mess). 1–dimensional boundaries have been studied by Kapovich and Kleiner. From their work it follows that if we can prove that w.o.p. a random group does not split, then w.o.p. its boundary is the Menger curve.

In fact, at density $d < \frac{1}{24}$, it is known that w.o.p. the boundary of a random group is the Menger curve, by a direct approach of Champetier. Moreover, from Żuk's theorem it follows that a random group $G$ at density greater than $\frac{1}{3}$ satisfies w.o.p. Kazhdan's property (T), hence it does not split and its boundary is also the Menger curve. We generalize this.

**Theorem.** *Let* $0 < d < \frac{1}{2}$*. Then with overwhelming probability, the boundary of a random group at density* $d$ *is the Menger curve.*

This a consequence of the following, which is our main theorem.

**Theorem.** *Let* $0 < d < 1$*. Then with overwhelming probability, a random group at density* $d$ *satisfies property* (FA)*, i.e.* $G$ *does not split as a free product with amalgamation and does not admit an epimorphism onto* $\mathbf{Z}$*.*