On finitely decomposable groups of automorphisms of a spherically homogeneous rooted tree.

Adam Woryna

Let $X = (X_0, X_1, \ldots)$ be an infinite sequence (the so called *changing alphabet*) of finite, nonempty sets X_i (*sets of letters*). For $i \geqslant 0$ let

$$X^*_{(i)} = \{x_0 x_1 \ldots x_s \colon x_j \in X_{j+i},\ 0 \leqslant j \leqslant s,\ s \geqslant 0\} \cup \{\epsilon\}$$

be a spherically homogeneous rooted tree of words (including the empty word ϵ) over the i-fold shift of X. We write

$$\mathcal{A}(i) = Aut(X^*_{(i)})$$

for the group of automorphism of the tree $X^*_{(i)}$. Any automorphism $g \in \mathcal{A}(i)$ acts by permutation on the first level of $X^*_{(i)}$, i.e. there is a homomorphism $\mathcal{A}(i) \ni g \mapsto \sigma^g \in Sym(X_i)$. Beside that any letter $x \in X_i$ defines an automorphism $g_x \in \mathcal{A}(i+1)$ by the equality $g(xw) = g(x)g_x(w),\ w \in X^*_{(i+1)}$. The mapping $g \mapsto ((g_x)_{x \in X_i}, \sigma^g)$ establishes an isomorphism (the so-called *wreath recursion*):

$$\mathcal{A}(i) \simeq \mathcal{A}(i+1) \wr Sym(X_i). \tag{1}$$

Definition 1 An automorphism $g \in \mathcal{A}(0)$ is *n-decomposable* if there are n-generated subgroups $H(i) \subseteq \mathcal{A}(i)$ $(i \geqslant 0)$ such that $g \in H(0)$ and the wreath recursion (1) induces an embedding

$$H(i) \hookrightarrow H(i+1) \wr Sym(X_i) \tag{2}$$

for every $i \geqslant 0$. An automorphism $g \in \mathcal{A}(0)$ is *finitely decomposable* if g is n-decomposable for some n. A group $G \subseteq \mathcal{A}(0)$ is *n-decomposable* if every its element is n-decomposable. A group $G \subseteq \mathcal{A}(0)$ is *finitely decomposable* if G is n-decomposable for some n.

Proposition 1 *The set $\mathcal{FD} \subseteq \mathcal{A}(0)$ of finitely decomposable automorphisms is an uncountable, proper subgroup of $\mathcal{A}(0)$ and every finitely generated group $G \subseteq \mathcal{FD}$ is finitely decomposable.*

Theorem 1 *For every finitely generated residually finite group G there is a changing alphabet X such that the group $\mathcal{A}(0) = Aut(X^*_{(0)})$ contains a 1-decomposable subgroup isomorphic to G.*

We present some constructions of finitely decomposable groups $G \subseteq \mathcal{A}(0)$ and describe their properties.

References

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