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Tame Words and Band-Systems for the Hawaiian Earrings and for Griffiths' space

[joint research with Oleg Bogopolski, Novosibirsk(Russia) and Düsseldorf (Germany)]

The Hawaiian Earrings Z are a metric one-point-union of circles with a common tangent point whose radii are given by a null-sequence. Although the space looks like a graph, its fundamental group is known not to be free. It does not possess a universal covering space in the classical sense, but in a generalized sense. In a prior result the action on this covering space was used to show that Rips' Theorem cannot be extended to groups that are not finitely generated. The talk will focus on a more recent result (joint with Oleg Bogopolski), where the goal was to understand better the fundamental group of Griffith's space Y (the one-point union of two cones over two distinct Hawaiian Earrings). This group can easily be seen as a quotient group of $\pi_1(Z)$, and the normal generators of the kernel of $\pi_1(Z) \to \pi_(Y)$ are also evident. However, it needed a better understanding of the general element inside this kernel, and the proof of a lemma that relates the factoring out of this kernel to combinatorial properties, to be able to prove the main results of this research project.

The fundamental group of the Hawaiian Earrings can be described by appropriate infinite combinatorial objects. The talk will introduce "tame words" for its description as a concept that is neither directly motivated by topology nor follows the idea to head at the utmost reduction of everything, but which turned out to be most useful for all our proofs. Based on this combinatorial description and on geometric considerations of band-systems and band-systems with infinite inscribed arch-systems, we were able to show the following:

- ▹ $\pi_1(Y)$ contains uncountably many elements which are infinite commutator products, but in spite of this, project to non-trivial elements in the homology group $H_1(Y)$.
- ▹ $\pi_1(Y)$ contains infinitely divisible elements and thus a subgroup isomorphic to $\mathbb{Q}$.
- ▹ $H_1(Y)$ contains an uncountable direct sum of such subgroups each of which is isomorphic to $\mathbb{Q}$.

In the first and third case we also recover by our proof-methods analogous known results for the Hawaiian Earrings.

The talk will try to explain the geometric and combinatoric ideas with which all the above mentioned results can be proven.