

**Antonio Avilés**

*Tukey classification of some sets arising in Banach spaces*

We look at the family of all weakly compact subsets of the ball of a separable Banach space. Assuming the axiom of analytic determinacy, we show that these families are classified into four categories under so-called Tukey equivalence. This means that the separable non-reflexive spaces are classified into three categories according to a certain degree of non-reflexivity. This is joint work with G. Plebanek and J. Rodriguez.

**Adel Babbah**

*On generalized Weyl's type theorem*

It is shown that if a bounded linear operator  $T$  or its adjoint  $T^*$  has the single-valued extension property, then generalized Browder's theorem holds for  $f(T)$  for every  $f \in \mathcal{H}(\sigma(T))$ . We establish the spectral theorem for the B-Weyl spectrum which generalizes a result in the Hilbert space case and we give necessary and sufficient conditions for such operator  $T$  to obey generalized Weyl's theorem.

**Artur Bartoszewicz**

*Topological and measure properties of some self-similar sets*

Given a finite subset  $\Sigma \subset \mathbb{R}$  and a positive real number  $q < 1$  we study topological and measure-theoretic properties of the self-similar set  $K(\Sigma; q) = \{ \sum_{n=0}^{\infty} a_n q^n : (a_n)_{n \in \omega} \in \Sigma^\omega \}$ , which is the unique compact solution of the equation  $K = \Sigma + qK$ . The obtained results are applied to studying partial sumsets  $E(x) = \{ \sum_{n=0}^{\infty} x_n \varepsilon_n : (\varepsilon_n)_{n \in \omega} \in \{0, 1\}^\omega \}$  of some (multigeometric) sequences  $x = (x_n)_{n \in \omega}$ .

For a finite subset  $\Sigma \subset \mathbb{R}$  of cardinality  $|\Sigma| \geq 2$ , we will write it as  $\Sigma = \{\sigma_1, \dots, \sigma_s\}$  for real numbers  $\sigma_1 < \dots < \sigma_s$ . Then we denote

$$\text{diam}(\Sigma) = \sigma_s - \sigma_1, \quad \delta(\Sigma) = \min_{i < s} (\sigma_{i+1} - \sigma_i), \quad \text{and} \quad \Delta(\Sigma) = \max_{i < s} (\sigma_{i+1} - \sigma_i).$$

Also put

$$I(\Sigma) = \frac{\Delta(\Sigma)}{\Delta(\Sigma) + \text{diam} \Sigma} \quad \text{and} \quad i(\Sigma) = \inf \{ I(B) : B \subset \Sigma, \quad 2 \leq |B| \}.$$

The self-similar sets  $K(\Sigma; q)$  where  $q \in (0, 1)$  have the following properties:

1.  $K(\Sigma; q)$  is an interval if and only if  $q \geq I(\Sigma)$ ;
2.  $K(\Sigma; q)$  is not a finite union of intervals if  $q < I(\Sigma)$  and  $\Delta(\Sigma) \in \{\sigma_2 - \sigma_1, \sigma_s - \sigma_{s-1}\}$ ;
3.  $K(\Sigma; q)$  contains an interval if  $q \geq i(\Sigma)$ ;
4. If  $d = \frac{\delta(\Sigma)}{\text{diam}(\Sigma)} < \frac{1}{3+2\sqrt{2}}$  and  $\frac{1}{|\Sigma|} < \frac{\sqrt{d}}{1+\sqrt{d}}$ , then for almost all  $q \in (\frac{1}{|\Sigma|}, \frac{\sqrt{d}}{1+\sqrt{d}})$  the set  $K(\Sigma; q)$  has positive Lebesgue measure and the set  $K(\Sigma; \sqrt{q})$  contains an interval;

5.  $K(\Sigma; q)$  is a Cantor set of zero Lebesgue measure if  $q < \frac{1}{|\Sigma|}$  or, more generally, if  $q^n < \frac{1}{|\Sigma_n|}$  for some  $n \in \mathbb{N}$  where  $\Sigma_n = \{ \sum_{k=0}^{n-1} a_k q^k : (a_k)_{k=0}^{n-1} \in \Sigma^n \}$ .
6. If  $\Sigma \supset \{a, a+1, b+1, c+1, b+|\Sigma|, c+|\Sigma|\}$  for some real numbers  $a, b, c \in \mathbb{R}$  with  $b \neq c$ , then there is a strictly decreasing sequence  $(q_n)_{n \in \omega}$  with  $\lim_{n \rightarrow \infty} q_n = \frac{1}{|\Sigma|}$  such that the sets  $K(\Sigma; q_n)$  has Lebesgue measure zero.

**Sudeshna Basu**

*Ball Separation properties in Banach spaces*

In this work, we study certain stability results for Ball Separation Properties in Banach Spaces leading to a discussion in the context of operator spaces. In this work, we study certain stability results for Small Combination of Slices Property (SCSP) leading to a discussion on SCSP in the context of operator spaces. SCS points were first introduced in [GGMS] as a “slice generalisation” of the PC (i.e. point of continuity points for which the identity mapping from weak topology to norm topology is continuous.) It was proved in [GGMS] that  $X$  is strongly regular (respectively  $X^*$  is  $w^*$  strongly regular) if and only if every non empty bounded convex set  $K$  in  $X$  (respectively  $K$  in  $X^*$ ) is contained in the norm closure (respectively  $w^*$ -closure) of  $\text{SCS}(K)$  (respectively  $w^*$ - $\text{SCS}(K)$ ) i.e. the SCS points ( $w^*$ -SCS points) of  $K$ . Later, it was proved in [S] that a Banach space has Radon Nikodym Property (RNP) if and only if it is strongly regular and it has the Krein-Milman Property (KMP). Subsequently, the concepts of SCS points was used in [R] to investigate the structure of non dentable closed bounded convex sets in Banach spaces. The ”point version” of the result in [S] (i.e. characterisation of RNP), was also shown to be true in [HL] which extends the results in [LLT].

#### References

- [B1] S. Basu, *The Asymptotic Norming Properties and related themes in Banach spaces*, Ph.D. Thesis, ISI, Calcutta (1998).
- [B2] *The ball generated property in operator spaces*, Indag. Math. **[13]** (2002), 169–175.
- [BB] S. Basu and P. Bandyopadhyaya, *On nicely smooth Banach spaces*, Extrac. Math., **[16]** (2001), 27–45.
- [HL] Z. Hu and B. L. Lin, *RNP and CPCP in Lebesgue-Bochner Function Spaces*, Illinois Journal of Mathematics **[37]** (1993), 329–347.
- [CL] D. Chen and B. L. Lin, *Ball Topology of Banach spaces*, Houston J. Math., **[22]** (1996), 821–833.
- [GGMS] N. Ghoussoub, G. Godefroy, B. Maurey and W. Schachermeyer, *Some topological and geometric structures in Banach spaces*, Mem. Amer. Math. Soc., **[378]** (1987).
- [LLT] B. L. Lin, P. K. Lin and S. Troyanski, *Characterisation of denting points*, Proc. Amer. Math. Soc. **[102]** (1988), 526–528.
- [R] H. P. Rosenthal, *On the structure of non-dentable closed bounded convex sets*, Adv. in Math., **[70]** (1988), 1–58.
- [S] W. Schachermeyer, *The Radon Nikodym Property are equivalent for strongly regular sets*, Trans. Amer. Math. Soc. **303** (1987), 268–316.

**Oscar Blasco**

*Fourier Analysis for vector measures*

We analyze the notion of convolution and Fourier transform for regular vector-valued measures defined on the borel sets of a compact abelian group. Special emphasis is made in the Riemann-Lebesgue lemma and Young's convolution type results in this setting. Applications to the general theory to the notions of Fourier transform and convolutions for functions in  $L^p(\nu)$  considered in [CGNS] are given. Also a weaker notion than the "norm integral translation invariance" of the measure  $\nu$ , considered in [DM], is introduced and showed to be good enough to extend some of the previous results achieved for functions in  $L^p(\nu)$ .

**References**

- [CGNS] Calabuig, J.M.; Galaz-Fontes, F.; Navarrete, E.M.; Sanchez-Perez, E. A. *Fourier transforms and convolutions on  $L^p$  of a vector measure on a compact Hausdorff abelian group*, J. Fourier. Anal. Appl. **19** (2013), 312-332.
- [DM] Delgado, O., Miana, P. *Algebra structure for  $L^p$  of a vector measure*, J. Math. Anal. Appl. **358** (2009), 355-563.

**Domenico Candeloro**

*Filter convergence and decompositions for vector lattice-valued measures*

Filter convergence of vector lattice-valued measures is considered, in order to deduce theorems of convergence for their decompositions, similar to the results obtained in [BC]. First the  $\sigma$ -additive case is studied, without particular assumptions on the filter; later the finitely additive case is faced, for measures defined on a  $\sigma$ -algebra, first assuming uniform  $s$ -boundedness then relaxing this condition but imposing some *typical* restrictions on the filter: for this part we make use of Schur-type theorems previously obtained in [ACKL, BDP]. Finally, strenghtening once more the conditions on the filter, but assuming convergence only in an algebra with the (SCP) property, we obtain again similar results: here the technique is inspired at the paper [C].

**References**

- [ACKL] A. Aviles Lopez, B. Cascales Salinas, V. Kadets, A. Leonov,  *$l_1$ -theorem for filters*, J. Math. Phys. Anal. Geom. **3** (2009), 383-398.
- [BC] A. Boccuto, D. Candeloro, *Convergence and decompositions for  $l$ -group-valued set functions*, Commentationes Mathematicae, **44**, (1) (2004), 11-37.
- [BDP] A. Boccuto, X.Dimitriou, N. Papanastassiou, *Schur lemma and limit theorems in lattice groups with respect to filters*, Math. Slovaca **62** (6) (2012), 1145-1166.
- [C] D. Candeloro, *Sui teoremi di Vitali-Hahn-Saks, Dieudonné, e Nikodým*, Rend. Circ. Mat. Palermo, ser. VII, **8**, (1985), 439-445.

**Diana Caponetti**

*On modular function spaces*

We consider modular function spaces  $E_\rho$  as defined in the book of W.M. Kozłowski (Modular function spaces, New York, Dekker, Basel, 1988). We are concerned with some aspects of these spaces, in particular we show the admisibility.

**Paola Cavaliere**

*On the absolute continuity of non-additive measures and applications to function spaces*

We are concerned with the notions of absolute continuity in terms of 0-continuity, and of  $(\epsilon, \delta)$ -continuity, with respect to a finitely additive function taking values into a topological commutative group. A sufficient condition for their equivalence is established. The non-additive case of functions with values into Hausdorff topological spaces is also investigated. In addition, applications to the study of function spaces will be discussed.

**Guillermo Curbera**

*On Random Unconditional Convergence in Banach and function spaces*

We review the concept of RUC systems in Banach spaces introduced by P. Billard, S. Kwapien, A. Pelczyński, and Ch. Samuel. We consider RUC systems in function spaces. In particular, we discuss a remarkable result by P. G. Dodds, E. M. Semenov, and F. A. Sukochev regarding orthonormal uniformly bounded system in rearrangement invariant spaces. We present an extension of this last result.

Joint work with Sergey V. Astashkin (University of Samara, Russia) and Konstantin E. Tikhomirov (University of Alberta, Canada).

**Harold Garth Dales**

*Multi-norms*

I shall discuss the theory of multi-norms. This has connections with norms on tensor products and with absolutely summing operators. There are many examples, some of which will be exhibited. In particular we shall describe multi-norms based on Banach lattices, define multi-bounded operators, and explain their connections with regular operators on lattices. We have new results on the equivalences of multi-norms, and a new representation theory. The theory is based on joint work with Maxim Polyakov (deceased), Hung Le Pham (Wellington), Matt Daws (Leeds), Paul Ramsden (Leeds), Oscar Blasco (Valencia), Niels Laustsen (Lancaster), and Vladimir Troitsky (Edmonton).

**Luisa Di Piazza**

*Non-absolute gage integrals for multifunctions with values in an arbitrary Banach space*

There is a rich literature dealing with Aumann, Debreu or Pettis integration of Banach-valued multifunctions. Such definitions involve in some way the Lebesgue integrability of the support functions or of the selections. Here we consider processes of integration for multifunctions which involves the notion of Henstock-Kurzweil integral of functions. This last integral has been introduced in 1957 by Kurzweil and, independently, in 1963 by Henstock: it is a simple modification of Riemann's method, and produces an integral more general than that of Lebesgue, and, at the same time, has the power of Lebesgue's one. Moreover it allows to integrate highly oscillating functions.

In this lecture we present some recent results about the Henstock-Kurzweil-Pettis and the Henstock integral for multifunctions taking values in an arbitrary Banach space.

In particular we discuss the special role played in the theory of Henstock-Kurzweil-Pettis integrable multifunctions by weakly sequentially complete Banach spaces and by spaces possessing the Schur property. We also prove that Henstock (and McShane) integrable multifunctions

possess Henstock (and McShane respectively) integrable selections. As an application we study the relationship among Henstock, McShane and Pettis integrals for multifunctions.

#### References

- [DM1] L. Di Piazza and K. Musiał, *Henstock-Kurzweil-Pettis integrability of compact valued multifunctions with values in an arbitrary Banach space*, Jour. Math. Anal. Applic. Vol. 408 (2013), pp. 452-464, ISSN: 0022-247X
- [DM2] L. Di Piazza and K. Musiał, *Relations among Henstock, McShane and Pettis integrals for multifunctions with compact convex values*, Monatshefte für Mathematik, Vol. 173, Issue 4 (2014), pp. 459-470, DOI: 10.1007/s00605-013-0594-y.

#### Piotr Drygier

##### *A class of hereditarily separably Sobczyk spaces*

We are concerned in the following problem: for what compact spaces  $K$  every copy of  $c_0$  in a space of continuous functions  $C(K)$  is complemented. In this paper we shall show that if compact space  $K$  satisfies certain measure-theoretic property (satisfied by for example Corson compacta, Rosenthal compacta) then every copy of  $c_0$  embeded in  $C(K)$  contains a subspace isomorphic to  $c_0$  complemented in  $C(K)$ .

#### Marián Fabian

##### *On coincidence of Pettis and McShane integrability*

It is well known that McShane integrable vector functions are Pettis integrable. According to R. Deville and J. Rodriguez, the converse is true if the target space is Hilbert generated, equivalently has Gateaux smooth norm. A. Avilés, G. Plebanek and J. Rodriguez constructed a weakly compactly generated, non-Hilbert generated space, of form  $C(K)$ , where the converse is not true. In the lecture we intend to study this phenomenon in more detail. We consider several concrete Eberlein (Corson) compact spaces  $K$  which are not uniform Eberlein from the perspective of possible coincidence/difference of Pettis and McShane integrability for functions from  $[0, 1]$  into  $C(K)$ . We actually focus just on the (equivalent) question if all scalarly negligible functions  $f : [0, 1] \rightarrow C(K)$  are McShane integrable.

#### Włodzimierz Fechner

##### *Abstract versions of the Radon-Nikodym theorem*

Abstract versions of the Radon-Nikodym theorem were obtained firstly by Dorothy Maharam [M1, M2]. She dealt with function-valued measures and abstract integrals. Her results were generalized in the framework of Riesz spaces by Wilhelmus A. J. Luxemburg and Anton R. Schep [LS, Theorems 3.1, 3.4 and 4.2].

In 1984 Wolfgang Arendt [A] proved factorization theorems for positive operators which provide a far-reaching generalization of some cases of the Luxemburg-Schep theorem.

Our purpose is to provide an extension of results of Arendt. We present factorization theorems, called by us factorization theorems of Arendt type, for additive monotone mappings defined on a lattice-ordered Abelian group and having values in a Dedekind complete Riesz space.

#### References

- [A] Wolfgang Arendt, *Factorization by lattice homomorphisms*, Math. Z. **185** (1984), 567–571.

- [F] Włodzimierz Fechner, *Factorization theorems of Arendt type for additive monotone mappings*, Non-linear Anal. **97** 2014, 138–144.
- [LS] W.A.J. Luxemburg and A.R. Schep, *A Radon-Nikodym type theorem for positive operators and a dual*, Nederl. Akad. Wet., Proc. Ser. A **81** (1978), 357–375.
- [M1] Dorothy Maharam, *The representation of abstract integrals*, Trans. Amer. Math. Soc. **75** (1953), 154–184.
- [M2] Dorothy Maharam, *On kernel representation of linear operators*, Trans. Amer. Math. Soc. **79** (1955), 229–255.

## Małgorzata Filipczak

### *Nonseparable spaceability and strong algebrability of sets of continuous singular functions*

A continuous function  $f: [a, b] \rightarrow \mathbb{R}$  of bounded variation ( $f \in CBV$ ) is said to be *singular* whenever it is not constant and  $f' = 0$ ,  $\lambda$ -almost everywhere. We examine the spaceability of some families of singular functions contained in the Banach algebra  $CBV$ , endowed with the norm

$$\|f\| = |f(0)| + Var(f)$$

where  $Var(f) = Var_{[0,1]}f$  denotes the total variation of  $f$  in  $[0, 1]$ .

A singular function  $f \in CBV$  is called *strongly singular* whenever its restriction to every subinterval of  $[0, 1]$  is singular. We show that the set of strongly singular functions in  $CBV$  is nonseparably spaceable that is it contains an infinite dimensional closed vector space.

We also prove that the set of strongly singular functions in  $CBV$  is strongly  $\mathfrak{c}$ -algebrable sets i.e. there is a set of continuum free generators of a subalgebra consisting of strongly singular functions. The argument is based on a general criterion of strong  $\mathfrak{c}$ -algebrability.

#### References

- [BBF] M. Balcerzak, A. Bartoszewicz, M. Filipczak, *Nonseparable spaceability and strong algebrability of sets of sets of continuous singular functions*, J. Math. Anal. Appl. **407** (2013), 263–269.

## David H. Fremlin

### *A martingale inequality*

The theory of stochastic integration is founded on the fact that we can define integration with respect to martingales even when they are not of bounded variation. The difficult cases are martingales which have jumps and are not  $L^2$ -processes. These are usually approached by means of the Fundamental Theorem of Martingales, which demands a deep understanding of continuous-time martingales and puts a number of restrictions on the context in which we can operate. I will outline an alternative approach depending on a kind of maximal inequality for finite martingales which seems to be new.

**Ondřej Kalenda***Quantitative Schur and Dunford-Pettis properties*

I will focus on recent quantitative strengthenings of the Schur theorem. First, the dual of any Banach space isometric to a subspace of  $c_0(\Gamma)$  has the strongest possible quantitative Schur property. Further, I will discuss limits to extend this result to subspaces of  $C(K)$  with  $K$  scattered and relationship to the two quantitative versions of the Dunford-Pettis property. The presented results are a joint work with Jiří Spurný.

**Sokol Bush Kaliaj***Descriptive Characterizations of Pettis and Bochner Integrals on  $m$ -dimensional compact intervals*

We consider the interval functions defined on the family  $\mathcal{I}$  of all non-degenerate closed subintervals of the unit interval  $I_0 = [0, 1]^m$  in the Euclidean space  $\mathbb{R}^m$  and taking values in a Banach space  $X$ . We give necessary and sufficient conditions for an additive interval function  $F : \mathcal{I} \rightarrow X$  to be the primitive of a Pettis or Bochner integrable function  $f : I_0 \rightarrow X$  in terms of the cubic average range of  $F$ .

**Tomasz Kania***Which Banach algebras of operators are Grothendieck spaces?*

We are concerned with the question asking for which reflexive Banach spaces  $E$ ,  $\mathcal{B}(E)$ , the Banach algebra of all bounded linear operators on  $E$  is a Grothendieck space. (A Banach space  $X$  is *Grothendieck* if weakly\* convergent sequences in  $X^*$  converge weakly.) This problem has its roots in Akemann's work on conditional expectations from von Neumann algebras. Pfitzner proved that von Neumann algebras (hence  $\mathcal{B}(H)$ , where  $H$  is a Hilbert space) are Grothendieck spaces. Diestel and Uhl wrote in their famous monograph: *Finally, there is some evidence that the space of bounded linear operators on a Hilbert space is a Grothendieck space and that more generally the space  $\mathcal{L}(X; X)$  [of all bounded operators on  $X$ ] is a Grothendieck space for any reflexive Banach space  $X$ .* We give a simple counter-example to the second part of this question: for  $E = (\bigoplus_n \ell_1^n)_{\ell_2}$ , we construct a complemented copy of  $\ell_1$  in  $\mathcal{B}(E)$ , which proves that  $\mathcal{B}(E)$  is not Grothendieck. Furthermore, we discuss the connection between the problem of existence of reflexive analogues of the Argyros–Haydon space and the Grothendieck property of  $\mathcal{B}(E)$ .

**Tomasz Kochanek***Stability of vector measures and non-trivial twisted sums of  $c_0$* 

We will deal with the Ulam-type stability problem for vector measures, that is, the question to what extent an analogue of the Kalton–Roberts theorem on nearly additive set functions (*Trans. Amer. Math. Soc.* 278 (1983), 803–816) remains true for vector-valued measures. We say that a Banach space  $X$  has the SVM (stability of vector measures) property, provided there is a constant  $v(X) < \infty$  such that for every set algebra  $\mathcal{F}$  and every function  $\nu : \mathcal{F} \rightarrow X$  satisfying

$$\|\nu(A \cup B) - \nu(A) - \nu(B)\| \leq 1 \quad \text{for all } A, B \in \mathcal{F} \text{ with } A \cap B = \emptyset,$$

there exists a (finitely additive) vector measure  $\mu : \mathcal{F} \rightarrow X$  satisfying  $\|\mu(A) - \nu(A)\| \leq v(X)$  for each  $A \in \mathcal{F}$ . We will report several results related to this notion, among which there are

characterizations of the SVM property in terms of twisted sums formed by the given space (or its dual). Then, we shall explain how such results may be used in order to construct new non-trivial twisted sums of classical Banach spaces: we strengthen the result obtained by Cabello Sánchez and Castillo (*Houston J. Math.* 30 (2004), 523–536) which says that there exists a non-trivial locally convex twisted sum of  $\ell_2$  and  $c_0$ , that is,  $\text{Ext}(c_0, \ell_2) \neq 0$ . Namely, we prove that in fact  $\text{Ext}(c_0, X) \neq 0$  for any infinite-dimensional reflexive sequence Banach space  $X$ . (Here, ‘sequence’ means that  $X$  is a linear subspace of  $\mathbb{R}^{\mathbb{N}}$  and the canonical unit vectors  $(e_n)_{n=1}^{\infty}$  form a 1-unconditional basis of  $X$ .)

### **Wiesław Kubiś**

*Universal operators between separable Banach spaces*

We describe two constructions of a non-expansive linear operator  $U$  acting between separable Banach spaces, such that its restrictions to closed subspaces form, up to isometries, the class of all non-expansive operators of separable Banach spaces.

### **Iwo Labuda**

*On a 1972 problem of Erik Thomas*

The title refers to a problem in the Thomas paper ‘On Radon maps with values in arbitrary topological vector spaces, and their integral extensions’ in which integration of scalar functions with respect to a vector measure was considered. In today’s terminology the problem can be stated as follows. Suppose the range space  $X$  of a vector measure does not contain any isomorphic copy of  $c_0$  (Banach space of sequences convergent to 0). Does it follow that the corresponding space of integrable functions does not contain  $c_0$  either? The range space  $X$  referred to above is an arbitrary sequentially complete Hausdorff topological vector space.

### **Niels Jakob Laustsen**

*Ideals of operators on the Banach space of continuous functions on the first uncountable ordinal*

I shall report on joint work with Tomasz Kania and Piotr Koszmider (IMPAN, Warsaw), in which we study the lattice of closed ideals of the Banach algebra  $\mathcal{B}(C_0[0, \omega_1])$  of bounded operators acting on the Banach space  $C_0[0, \omega_1]$  of scalar-valued, continuous functions which are defined on the locally compact ordinal interval  $[0, \omega_1]$  and vanish eventually. (Here  $\omega_1$  denotes the first uncountable ordinal.) Our main theorem gives a number of equivalent conditions, each describing the unique maximal ideal  $\mathcal{M}$  of  $\mathcal{B}(C_0[0, \omega_1])$ . Among the consequences of this result are that  $\mathcal{M}$  has a bounded left approximate identity (this complements a 25-year old result of Loy and Willis stating that  $\mathcal{M}$  has a bounded right approximate identity) and that  $\mathcal{B}(C_0[0, \omega_1])$  has a unique second-largest ideal.

### **Zbigniew Lipecki**

*The variation and semivariations of a vector measure*

This is a survey of the speaker’s results (2003, 2004, 2010) on the variation and semivariations of a vector measure. They have origin in previous work of R. G. Bartle, N. Dunford and J. T. Schwartz (1955), E. Thomas (1974), L. Janicka and N. J. Kalton (1977), R. Anantharaman and K. M. Garg (1983), L. Drewnowski and Z. Lipecki (1995) as well as G. G. Lorentz (1952) and G. P. Curbera (1994). By a vector measure we mean a  $\sigma$ -additive function on a  $\sigma$ -algebra



$\Sigma$  of subsets of some set with values in a Banach space  $X$ . Let  $\nu$  be a (positive) measure on  $\Sigma$ , and set

$$\mathcal{E}_\nu(X) = \{\varphi \in \text{ca}(\Sigma, X) : |\varphi| = \nu\}.$$

We present necessary and sufficient conditions on  $\nu$  that  $\mathcal{E}_\nu(X)$  be nonempty for some  $X$ , i.e., we characterize  $|\varphi|$  as a measure on  $\Sigma$ . As a by-product, we show that if  $\mathcal{E}_\nu(X)$  is nonempty, then so is  $\mathcal{E}_\nu(c_0)$ . Thus,  $c_0$  is "variation universal" in the class of Banach spaces. We also discuss the Borel complexity of  $\mathcal{E}_\nu(X)$  in  $\text{ca}(\Sigma, X)$  as well as its denseness in an appropriate subspace of this space. Finally, we characterize the semivariations  $\tilde{\varphi}$  (of Bartle, Dunford and Schwartz) and  $\bar{\varphi}$  as order continuous submeasures that are multiply subadditive in the sense of Lorentz. In addition, we show that  $c_0$  is universal for the former semivariation when we restrict attention to separable nonatomic vector measures, which is also a corollary to an earlier result of Curbera.

### **Witold Marciszewski**

#### *Remarks on extension operators*

Given a compact space  $K$ , by  $C(K)$  we denote the Banach space of continuous real-valued functions on  $K$ , equipped with the standard supremum norm. For a closed subspace  $L$  of  $K$ , a bounded linear operator  $T : C(L) \rightarrow C(K)$  is called an extension operator if, for every  $f \in C(L)$ ,  $Tf$  is an extension of  $f$ .

We show that if  $B_H$  is the unit ball of a Hilbert space  $H$  of density greater or equal  $\omega_\omega$ , equipped with the weak topology, then, for any  $0 < \lambda < \mu$ , there is no extension operator  $T : C(\lambda B_H) \rightarrow C(\mu B_H)$ .

### **Valeria Marraffa**

#### *Quadratic Variation of Martingales in Riesz Spaces*

It is possible to extend aspects of discrete-time stochastic processes in the classical probability space setting to Riesz spaces. In this approach the order theory replaces the measure theory. In the measure-free setting of Riesz spaces, the quadratic variation of martingales, sub- and supermartingales is considered and quadratic variation inequalities for discrete-time processes are derived. In particular, an analogue of Austin's sample function theorem on convergence of the quadratic variation process of a martingale is proved.

### **Mieczysław Mastyło**

#### *Factorization theorems for multilinear operators*

We will discuss some recent works with Enrique A. Sánchez Pérez, concerning the domination and factorization theorems for multilinear operators. In particular we will present variants of Pisier's factorization theorem for summing multilinear operators.

### **Kazimierz Musiał**

#### *Fuzzy Integration*

I will present basic notions concerning fuzzy integration of fuzzy number valued functions with respect to the Lebesgue measure. Then, I will sketch a proof of a decomposition theorem for the fuzzy Henstock integral.

**Kirill Naralnikov**

*A Lusin type measurability property for vector-valued functions*

We study the class of ‘Riemann measurable’ vector-valued functions based on a Lusin type property. This class contains all Riemann integrable functions and is closely related to the restricted versions of the McShane and Henstock integrals, the  $\mathcal{M}$ - and  $\mathcal{H}$ -integrals, defined by means of Lebesgue measurable gauges.

**Martin Rmoutil**

*Norm-attaining functionals and proximal subspaces*

For a non-reflexive Banach space  $X$  and its closed subspace  $Y \subset X$  of finite codimension in  $X$ , consider the following two sentences:

1.  $Y$  is proximal in  $X$  (i.e. for each  $x \in X$  there is  $y \in Y$  such that  $\|x - y\| = \text{dist}(x, Y)$ );
2.  $Y^\perp \subset \text{NA}(X) := \{x^* \in X^*; \exists x \in B_X : x^*(x) = \|x^*\|\}$ .

It is easy to prove (1)  $\implies$  (2) for any  $X$ . We show that for some Banach spaces  $X$  the opposite implication holds, and for some it does not (we construct counterexamples).

Very recently Charles J. Read constructed a renorming  $\|\cdot\|$  of  $c_0$  such that  $(c_0, \|\cdot\|)$  contains no proximal subspaces of finite codimension greater than 1. By proving the equivalence of (1) and (2) for the space  $(c_0, \|\cdot\|)$  we also provide a negative solution to the long-standing problem of G. Godefroy of 2-lineability of  $\text{NA}(X)$  (that is, whether the set  $\text{NA}(X)$  always contains a two-dimensional subspace).

#### References

- [G] G. Godefroy, *The Banach Space  $c_0$* , Extracta Math. **16** (1), 1–25, 2001.
- [GI] G. Godefroy, V. Indumathi, *Proximality in subspaces of  $c_0$* , J. Approx. Theory **106**, 175–181, 1999.
- [R] C.J. Read, *Banach spaces with no proximal subspaces of codimension 2*, preprint

**Jose Rodriguez**

*Compactness in  $L^1$  of a vector measure*

We will deal with compactness in  $L^1$  spaces of vector measures. The weak topology and the topologies associated to the convergence of the integrals will be considered. Joint work with J.M. Calabuig, S. Lajara and E.A. Sánchez-Pérez.

**Anna Rita Sambucini**

*On set-valued Henstock–Mc Shane integral in Banach (lattice) space setting*

We study Henstock-type integrals for functions defined in a Radon measure space and taking values in a Banach lattice  $X$ . Both the single-valued case and the multivalued one are considered (in the last case mainly  $cwk(X)$ -valued mappings are discussed) and a comparison with the Aumann integral is given.

In the single-valued case, we obtain comparisons between the norm- and the order-type integral, which become more interesting when  $X$  is an  $L$ -space.

In the multivalued case, by using an embedding theorem, we deduce the norm-integral to that of a single-valued function taking values in an  $M$ -space: in this way we easily obtain new proofs for some decomposition results recently obtained.

Also the order-type integral for multivalued mappings has been studied: a previous definition ([BMS]) is restated in an equivalent way, some selection theorems are stated and a decomposition of the previous type is deduced also in this setting.

#### References

[BMS] A. Boccuto - A.M. Minotti - A.R. Sambucini, *Set-valued Kurzweil-Henstock integral in Riesz space setting*, PanAmerican Mathematical Journal **23** (1) (2013), 57–74.

#### Damian Sobota

*On Corson's property (C) and Maharam type of measures*

I will present some relations between the property (C) of Corson in the space  $C(K)$  for a given compact space  $K$  and the tightness of the space  $P(K)$  of regular probability measures on  $K$  endowed with the *weak\** topology. Investigating the Maharam type of measures on  $K$ , I will show the following equivalence: the space  $C(K \times K)$  has property (C) if and only if  $P(K \times K)$  has countable tightness.

#### Radosław Szwedek

*Eigenvalues and spectral properties of entropy numbers of operators*

We will discuss some recent works with Mieczysław Mastyło, concerning modern topics in the theory of operators on Banach spaces. We introduce and study entropy and spectral moduli of operators, and show relationships between these moduli and eigenvalues of operators. Combining our results with interpolation techniques yields an interpolation variant of the famous Carl-Triebel inequality.

#### Paolo Vitolo

*Openness of measures and closedness of their range*

It is known that every  $\mathbb{R}^n$ -valued  $\sigma$ -additive measure defined on a  $\sigma$ -algebra has a closed range. Moreover, in 1988, Spakowski proved that every nonatomic  $\sigma$ -additive measure on a  $\sigma$ -algebra is an open map.

We generalize these results to modular measures on D-lattices, proving that every  $\mathbb{R}^n$ -valued  $\sigma$ -order continuous modular measure  $\mu$  on a  $\sigma$ -complete D-lattice  $L$  has a closed range, and is an open map if  $L$  is  $\mu$ -chained.

Recall that D-lattices (otherwise called lattice-ordered effect algebras) are a common generalization of orthomodular lattices and MV-algebras. Hence the study of modular measures on D-lattices allows us to unify the study of modular functions on orthomodular lattices in the context of non-commutative measure theory and the study of measures on MV-algebras in the context of fuzzy measure theory.

**Witold Wnuk**

*On the algebraic sum of ideals and sublattices*

There are three types of Banach lattices: discrete (represented by sequence spaces), continuous (containing  $L^p(\mu)$  spaces over an atomless measure  $\mu$  and  $C(K)$  spaces on compacts  $K$  without isolated points) and heterogeneous (i.e., neither continuous nor discrete). We will discuss an old problem concerning closedness of the algebraic sum of a closed ideal and a closed sublattice. It occurs that for every closed infinite dimensional and infinite codimensional ideal  $I$  in a Banach lattice  $E$  there exists a closed discrete sublattice  $F \subset E$  such that  $I + F$  is not closed. We indicate several (not very restrictive) conditions implying that for a closed ideal  $I$  we can find a closed continuous (and a closed heterogeneous) sublattice  $F$  with  $I + F \neq \overline{I + F}$ .

**Wiesław Żelazko**

*Some Problems in Topological Algebras*

All considered algebras will be commutative, unital, and complete (with jointly continuous multiplication).

1. Problems concerning topologically invertible elements.

An element  $x \in A$  is said topologically invertible, if there is a net  $(z_\alpha) \subset A$  such that

$$\lim_{\alpha} z_{\alpha}x = e, \text{ the unity of } A.$$

Among some problems concerning this concept, I shall present an example of an inductive limit of some function algebras which is not a field and in which all non-zero elements are topologically invertible (a modification of a more complicated example given in [1]). I shall discuss the question, whether the multiplication in this example is jointly continuous and whether it is a complete space.

2. Two, over 50 years old, famous open problems:

- (a) The problem of continuity of characters (Michael-Mazur Problem and some of its versions).
- (b) Is the Gelfand-Mazur theorem true for F-algebras (completely metrizable algebras).

## References

- [1] A. Atzmon, *An operator without invariant subspaces on a nuclear Fréchet space*, Acta of Math. 117 (1983), 669-694.

**Mohamed Zohry**

*An example of a special set-valued integrably bounded function*

For a special class of integrably bounded closed-convex-nonempty valued random variable, it is shown that the conditional expectation behaves as a "centralizer" with respect to the multiplication by elements of  $L^\infty$ . This gives a type of a result of Hiai-Umegaki.