Equivalence of multi-norms

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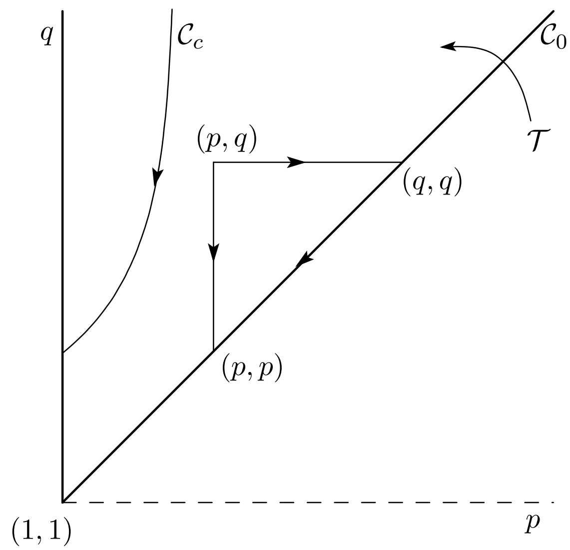
Thanks to Hung Le Pham for help with the pictures

Picture 1: The (p, q)-triangle



Picture 2: Larger/smaller (p, q)-multi-norms

Given a $(\bar{p}, \bar{q})$-multi-norm, the following figure illustrates the regions where the (p, q)-multinorms are definitely smaller and larger than this particular $(\bar{p}, \bar{q})$-multi-norm. We have not at this stage excluded the possibility that the shaded regions are larger.

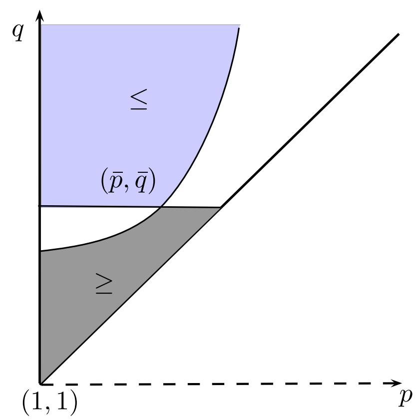


FIGURE 1. Regions where the (p, q)-multinorms are smaller and are larger than a particular $(\bar{p}, \bar{q})$-multi-norm.

Picture 3: The case where $r \geq 2$

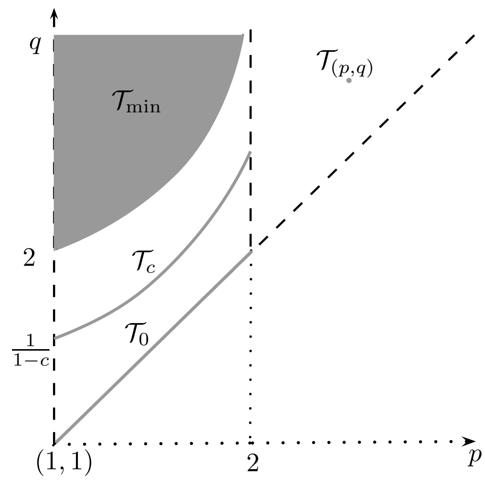


FIGURE 2. The various mutually disjoint equivalence classes for (p, q)-multi-norms on ℓ^r for $r \geq 2$.

Picture 4: The case where $1 < r < 2$

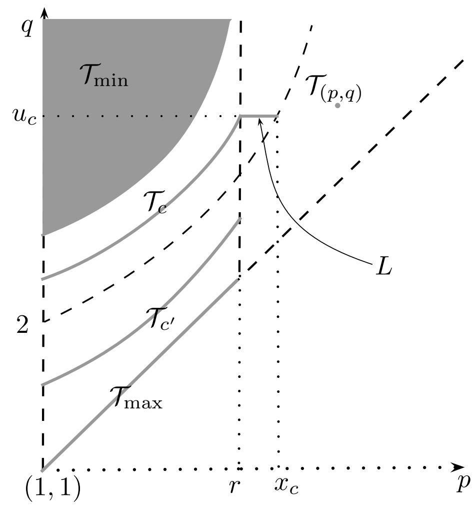


FIGURE 3. The various mutually disjoint equivalence classes for (p, q)-multi-norms on ℓ^r for $1 < r < 2$. We do not know if points on L are mutually equivalent, and we do not know whether any point of L is equivalent to any point of remaining part of the curve $\mathcal{T}_c$..

Picture 5: The set $B_{r,t}$

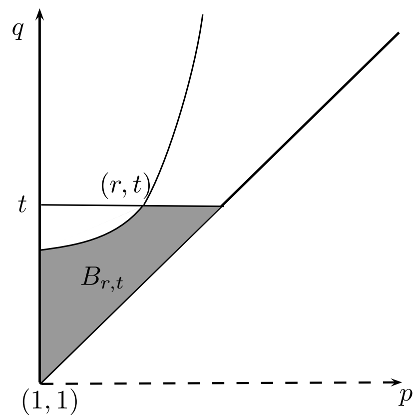


FIGURE 4. Here $1 \leq r \leq t$

Picture 6: The sets $B_{1,t}$ and $D_{1,t}$

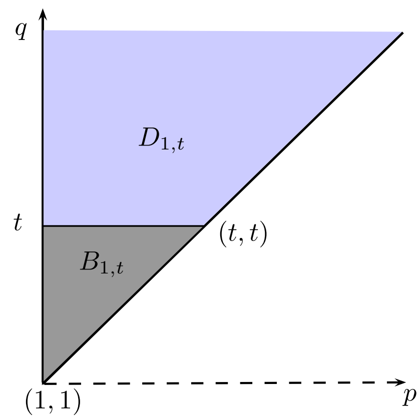


FIGURE 5. The set $B_{1,t}$ contains the line with $q = t$ for $1 \leq p \leq t$ and the set $D_{1,t}$ contains the line with $q = t$ for $1 \leq p < t$.

Picture 7: The sets $B_{r,t}$ and $D_{r,t}$ for $r \geq 2$

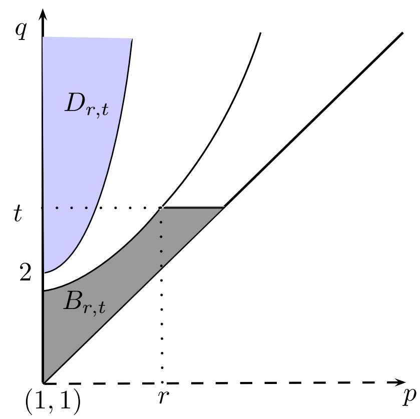


Figure 6. Here the two sets are disjoint, and so the standard t-multi-norm is not equivalent to any (p, q)-multi-norm.

Picture 8: The sets $B_{r,t}$ and $D_{r,t}$ for $r \geq 2$

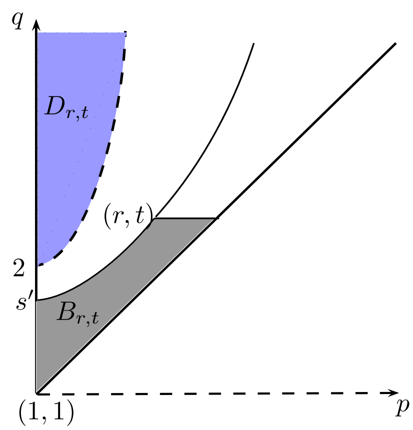


FIGURE 7. Here $1/r - 1/t \leq 1/2$.

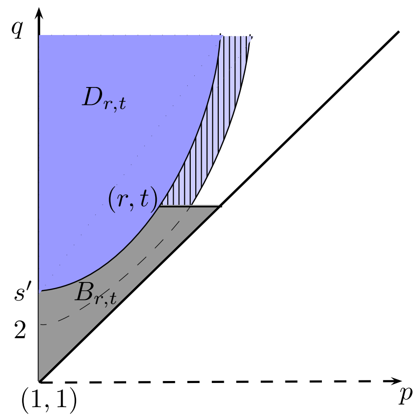


FIGURE 8. Here $1/s = 1/r - 1/t > 1/2$.

Here we do not know whether $D_{r,t}$ contains the shaded area