

Spectral properties of unbounded Jacobi matrices

Abstract

The aim of the dissertation is the understanding of spectral properties of unbounded Jacobi matrices. Properties of associated orthogonal polynomials and the corresponding measure of orthogonality of the polynomials are also studied.

The main motivation of the dissertation is Chihara's problem concerning spectral properties of a class of orthogonal polynomials. The problem is motivated by a result of Chihara obtained in 1968. In several subsequent articles the problem was mentioned but the explicit formulation comes from articles published in 1991 and 2003. Until now the problem was open. In the first part of the dissertation there are given additional assumptions which imply the positive answer to the problem. Moreover, we give additional assumptions to guarantee the conclusion of a (false in general) conjecture due to Roehner-Valent from 1982 concerning spectral properties of birth and death processes. We also generalize results due to Janas, Moszyński and Pedersen. This part is accepted for publication in *Constructive Approximation*.

The second part concerns asymptotics of generalized eigenvectors. Obtained results via the theory of strong non-subordinacy allowed to prove, under some regularity assumptions that spectra of Jacobi matrices are absolutely continuous and to identify spectrum as a set. The results generalize and simplify the proofs of theorems obtained by other authors.

Finally, in the third part we show that in the most of the cases considered in the second part, one can show absolute continuity of spectrum in a completely constructive way. Specifically, we give a formula for the density of the measure of orthogonality of associated orthogonal polynomials. The formula implies that the density is a continuous positive function. Numerical examples are provided to demonstrate the possibility of approximation of the measure. Moreover, there was shown that under additional assumptions Christoffel functions are convergent. In this way we completely solve Ignjatović conjecture formulated in 2009.