

SYMMETRIC MOMENT PROBLEMS AND A CONJECTURE OF VALENT

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ABSTRACT. I shall report on joint work with Ryszard Szwarc related to the following:.

In 1998 G. Valent made conjectures about the order and type of certain indeterminate Stieltjes moment problems associated with birth and death processes having polynomial birth and death rates of degree $p \geq 3$. Romanov recently proved that the order is $1/p$ as conjectured. We prove that the type with respect to the order is related to certain multi-zeta values and that this type belongs to the interval

$$[\pi/(p \sin(\pi/p)), \pi/(p \sin(\pi/p) \cos(\pi/p))],$$

which also contains the conjectured value. This proves that the conjecture about type is asymptotically correct as $p \rightarrow \infty$.

The main idea is to obtain estimates for order and type of symmetric indeterminate Hamburger moment problems when the orthonormal polynomials P_n and those of the second kind Q_n satisfy $P_{2n}^2(0) \sim c_1 n^{-1/\beta}$ and $Q_{2n-1}^2(0) \sim c_2 n^{-1/\alpha}$, where $0 < \alpha, \beta < 1$ can be different, and c_1, c_2 are positive constants. In this case the order of the moment problem is majorized by the harmonic mean of α, β . Here $\alpha_n \sim \beta_n$ means that $\alpha_n/\beta_n \rightarrow 1$. This also leads to a new proof of Romanov's Theorem that the order is $1/p$.

REFERENCES

- [1] C. Berg and R. Szwarc, *On the order of indeterminate moment problems*, Advances in Mathematics **250** (2014), 105–143.
- [2] C. Berg and R. Szwarc, *Symmetric moment problems and a conjecture of Valent*. ArXiv:1509.06540. Mat. Sbornik **203** no. 3 (2017), 28–53. (russian). English version to appear.