Definable Valuations on Dependent Fields

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Abstract

In my PhD thesis I worked on finding model theoretic and field theoretic conditions, under which a field admits a definable valuation.

By a conjecture of Shelah, every strongly dependent infinite field, is either real closed, algebraically closed or "p-adic like. As the p-adic valuation is definable on the field of p-adic numbers, this motivates us to ask under which conditions not real closed, not algebraically closed, infinite, dependent fields, admit non-trivial definable valuations.

It is possible to show that (under some additional conditions) an infinite, non-algebraically closed and non-real closed field K, admits a non-trivial definable valuation, if for some prime $p \neq \mathrm{char}(K)$ the sets of the form $\bigcap_{i=1}^{n} a_i \cdot \left((K^\times)^p + 1\right)$ for $n \in \mathbb{N}$ and $a_i \in K^\times$ are a basis of zero neighbourhoods of a V-topology. This can be expressed by six axioms (V 1) to (V 6). It can be shown that for the given sets, the axioms (V 2) and (V 5) always hold and the remaining axioms can be considerably simplified. Under the assumption that K is dependent and $\left(K^\times : (K^\times)^q\right) < \infty$, we can further show that (V 1) holds. Currently our aim is to find conditions under which the simplified versions of the axioms (V 3), (V 4) and (V 6) hold.

In my talk I will give an overview over the results of my thesis.

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