INTRODUCTION TO STABLE GROUPS - PROBLEM LIST 2

Let G be a group definable in an ω-stable theory. Recall that for any stationary $p \in S(A) \cap [x \in G]$ we have defined $g \cdot p := \text{tp}(g \cdot a/A)$ for $a \models p$ such that $a \perp_A g$.

(1) Prove that $\text{RM}(g/A, h) = \text{RM}(g \cdot h/A, h)$ for any $g, h \in G$. Conclude that $\text{RM}(g \cdot p) = p$ for any stationary p and any $g \in G$.

(2) Prove that as $\text{Stab}_G(p) = \text{Stab}_G(\tilde{p})$ (recall $\tilde{p}$ is the unique global nonforking extension of p).

(3) Prove that $\text{Stab}_G(p) := \{g \in G : g \cdot p = p\}$ is definable. Hints: Use Problem 2 to assume WLOG that p is a global type. (ii) As p is stationary, $\text{DM}(p) = 1$, so there is a formula $\phi \in p$ with $\text{RM}(p) = \text{RM}(\phi)$ and $\text{DM}(\phi) = 1$. (iii) Use definability of types in stable theories.

(4) Let S be a definable group acting definably on G. Prove that if $A \subseteq G$ is S-invariant and such that $|A/H| > 1 \Rightarrow |A/H| \geq \omega$ for every S-invariant definable group $H \leq G$, then A is indecomposable. Hint: use the descending chain condition.