INTRODUCTION TO STABLE GROUPS - PROBLEM LIST 3

(1) Let G be any group and let $g \in G$. Prove that $|a^G| = [G : C_G(a)]$, where $a^G := \{g^{-1}ag : g \in G\}$ and $C_G(a) := \{g \in G : ga = ag\}$.

(2) Let G be any group and let H be its definable subgroup. Prove that $\mathrm{RM}(G) \geq \mathrm{RM}(H) + \mathrm{RM}(G/H)$. Hint: induction on $\mathrm{RM}(H)$ for a more general statement about definable surjections.

(3) Prove that if K is a field of finite Morley rank, then it has no proper infinite definable subring. Hint: Use that a stable semigroup is a group, and the Mactintyre's theorem (i.e. that infinite ω-stable fields are algebraically closed).

(4) Let K be a field of finite Morley rank with $char(K) = 0$. Prove that if $X \subseteq K$ is infinite and definable, then, for some $n < \omega$, each element of K is a sum of at most n elements of X. Hint: Use that K has no proper nontrivial definable additive subgroups and ZIT.