# INTRODUCTION TO STABLE GROUPS - PROBLEM LIST 4

(1) Let $A$ be an abelian group acting definably on an ablian group $B$. Let $R$ be the ring of endomorphisms of $B$ generated by the action of $A$ on $B$. Assume there are $n < \omega$ such that $R = \{a_1 + \cdots + a_n : a_1, \ldots, a_n \in A\}$ ($a_i$ here means the endomorphism of $B$ induced by $a_i$), and $b \in B$ such that $B = R \cdot b$. Prove that $R$ is interpretable.

(2) Prove that if $G$ is connected and $Z(G)$ is finite then $Z(G/Z(G)) = e$. Hint: use Problem 1 from List 3.

(3) Deduce from Reinecke Theorem (6.5) that every infinite $\omega$-stable group has an infinite definable abelian subgroup.

(4) Let $K, H \leq G$ be such that $K^h = K$ for every $h \in H$. Let $a \in G$ and put $H_a := \{x \in H : a^h \cdot K = a \cdot K\}$. Prove that $H_a$ is a subgroup of $H$.