Geometric and Asymptotic Group Theory II

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Blatt 4

(1) Show that the following groups are metabelian:
   (a) A dihedral group.
   (b) Heisenberg group $H_3(\mathbb{Z}) = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\}$.
   (c) The *lamplighter group* $\mathbb{Z}_2 \wr \mathbb{Z}$ (the wreath product of $\mathbb{Z}_2$ and $\mathbb{Z}$, i.e. $(\bigoplus_{\mathbb{Z}} \mathbb{Z}_2) \rtimes \mathbb{Z}$).

(2) Show that the following definitions of a metabelian group are equivalent.
   (a) G is metabelian if the commutator subgroup $[G, G]$ is abelian.
   (b) G is metabelian if it is solvable of degree 2.
   (c) G is metabelian if there exists a normal subgroup N of G such that both G/N and N are abelian.

(3) Show that the free product of two residually finite groups is residually finite.

(4) Show that any subgroup of a residually finite group is residually finite.