# Geometric and Asymptotic Group Theory II 

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http://www.mat.univie.ac.at/~dosaj/GGTWien/Course.html
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Blatt 5

## The proof of Britton's lemma

Let $\operatorname{BS}(1,2)=\left\langle a, t \mid t^{-1} a t=a^{2}\right\rangle$ act by "adjoining a generator on the left and reduction" on the set $X:=\left\{t^{i} a^{j} t^{-k} \mid k, i \geq 0, j \in \mathbb{Z}\right\} / \sim$.
(1) Compute the action of any finite subsequence of the sequence
$\cdots t^{-1} a^{p_{1}} t^{-1} a^{p_{2}} t^{-1} a^{p_{3}} \cdots$, for $p_{i} \in \mathbb{Z}$, on $1 \in X$ (i.e. $t^{i} a^{j} t^{-k}$ for $k=i=$ $j=0$ ).
(2) Compute the action of any finite subsequence of the sequence $t a^{2 r+1} t^{-1} a^{p_{1}} t^{-1} a^{p_{2}} t^{-1} a^{p_{3}} \cdots$ on $1 \in X$.
(3) Compute the action of any finite subsequence of the sequence $\cdots t a^{s_{3}} t a^{s_{2}} t a^{s_{1}} t a^{2 r+1} t^{-1} a^{p_{1}} t^{-1} a^{p_{2}} t^{-1} a^{p_{3}} \cdots$ on $1 \in X$.
(4) Conclude that an element of $\mathrm{BS}(1,2)$ represented by a reduced form is nontrivial.

