

## Geometric and Asymptotic Group Theory II

Damian Osajda

damian.osajda@univie.ac.at

<http://www.mat.univie.ac.at/~dosaj/GGTWien/Course.html>

Dienstag, 11:00–12:00, Raum C2.07 UZA 4

Blatt 6

### Amenability

- (1) Show that if  $F\partial_S G = 0$  wrt some finite generating set  $S$ , then  $F\partial_{S'} G = 0$  wrt any other finite generating set  $S'$  of  $G$ .
- (2) Show that if  $F\partial_S G = 0$  wrt one of the definitions of the boundary, then  $F\partial_S G = 0$  wrt any other boundary.
- (3) Show that the following two conditions are equivalent for a group  $G$  generated by a finite set  $S$ :
  - (a)  $\forall \epsilon \exists \text{ fin. } A \subseteq G \frac{|\partial_S A|}{|A|} < \epsilon$ ,
  - (b)  $\forall \epsilon \forall \text{ fin. } K \subseteq G \exists \text{ fin. } A \subseteq G \forall g \in K \frac{|gA\Delta A|}{|A|} < \epsilon$ .
- (4) Show that  $\mathbb{Z}^n$  is amenable.
- (5) Show that a subgroup of an amenable group is amenable.
- (6) Show that the direct product of two amenable groups is itself amenable.
- (7) Is the Baumslag-Solitar group  $BS(2,3) = \langle a, t \mid ta^2t^{-1}a^{-3} \rangle$  amenable?
- (8) Prove that the Baumslag-Solitar group  $BS(1,2) = \langle a, t \mid tat^{-1}a^{-2} \rangle$  is amenable.
- (9) Show that metabelian groups are amenable.