Wstęp do topologii algebraicznej

Ćwiczenia 2

(1) Let $f\colon D \to \mathbb{R}^n$ be a continuous map defined on a closed subset D of a normal space X. Show that there exists a continuous extension $F\colon X \to Y$ of f.
(2) Show that $\mathbb{R}$ is homeomorphic to $(0,1)$.
(3) Show that $[0,1]$ is compact, and $[0,1)$, $(0,1)$, $\mathbb{R}$ are not.
(4) Show that, for $k \geqslant 1$, $[0,1]^k$ is compact, and $[0,1)^k$, $(0,1)^k$, $\mathbb{R}^k$ are not.
(5) Show that the Cantor set is compact.
(6) Show that a subset of the Euclidean space is compact iff it is closed and bounded.
(7) Give an example of a continuous bijection not being a homeomorphism. Show that any continuous bijection $\mathbb{R} \to \mathbb{R}$ is a homeomorphism.
(8) Show that a compact subset of a Hausdorff space is closed.