Wstęp do topologii algebraicznej

Ćwiczenia 4

(1) Show that the formula $\sum_{i=1}^{\infty} \frac{1}{2^i}|x_i - y_i|$ defines a metric on $[0,1]^{\aleph_0}$. Show that the metric topology coincides with the Tichonov one.
(2) Let $\mathcal{B}$ be a countable basis of a regular space X. Let $(f_i)_{i=1}^{\infty}$ be the sequence of Urysohn functions for all admissible pairs of elements of $\mathcal{B}$. Show that the map into the Hilbert cube $X \to Q \colon x \mapsto (f_i(x))_{i=1}^{\infty}$ is an embedding.
(3) Are the following spaces homogeneous: S^1, S^n, $\mathbb{R}^n$, $\{(x,y) \in \mathbb{R}^2 \mid y \geqslant 0\}$?