Wstęp do topologii algebraicznej

Ćwiczenia 6

(1) Show that a continuous image of a connected space is connected.
(2) Show that:
   - $f \simeq f$,
   - $f \simeq g$ iff $g \simeq f$,
   - if $f \simeq g$ and $g \simeq h$ then $f \simeq h$.
(3) Show that a space X is contractible iff every map $f\colon X \to Y$, for arbitrary Y, is homotopic to a constant map.
(4) Show that X is contractible iff every map $f\colon Y \to X$ is nullhomotopic, that is, $[Y, X] = \{*\}$.
(5) Let X be a path-connected space. Show that any loop $f\colon S^1 \to X$ is homotopic to a loop based at a given $x_0 \in X$, that is, a loop with $f(1, 0) = x_0$. Show that two loops $f_1, f_2\colon S^1 \to X$ based at $x_0 \in X$ are homotopic iff they are homotopic through loops based at x_0.