Wstęp do topologii algebraicznej

Ćwiczenia 7

(1) Is a continuous image of a contractible space contractible? What about simply connected spaces?
(2) Find all coverings of a circle.
(3) Let $f\colon Y \to Z$ be a continuous map. Show that if $g\colon X \to Y$ is homotopic to a constant map then $f \circ g$ is null-homotopic as well.
(4) Show that the "identity" loop $S^1 \to S^2$ to the equator is null-homotopic.
(5) (*Compact-open topology*) Let X, Y be topological spaces. For every pair (K, U) consisting of a compactum $K \subseteq X$ and an open set $U \subseteq Y$ we define a subset $N_{K,U} = \{f \mid f(K) \subseteq U\}$ of X^Y. Show that the collection of $N_{K,U}$ forms a sub-base of a topology.
(6) Show that if X is compact and Y is a metric space then the compact-open topology is the same as the topology given by the supremum metric.