Grupy i kompleksy

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Ćwiczenia 3

(1) Show that if S, S' are two finite generating set for G, then for the corresponding word metrics $d_S, d_{S'}$, the metric spaces (G, d_S) and $(G, d_{S'})$ are bi-Lipschitz equivalent.

(2) Show that the map $f\colon (X, d_X) \to (Y, d_Y)$ is a coarse embedding if for every two sequences $(x_n) \subset X$, $(y_n) \subset Y$ we have $d_X(x_n, y_n) \to \infty$ iff $d_Y(f(x_n), f(y_n)) \to \infty$.

(3) Show that a coarsely surjective coarse embedding admits a coarse inverse.

(4) Show that a composition of quasi-isometric embeddings is a quasi-isometric embedding. Show that a composition of quasi-isometries is a quasi-isometry. Show that for a quasi-isometry there exists a quasi-inverse map.

(5) Find a quasi-isometric embedding of a half-line $\mathbb{R}^+ = [0, +\infty)$ into the Euclidean plane $\mathbb{E}^2$, which is not at a finite distance from a geodesic ray in $\mathbb{E}^2$. Can one find a similar embedding into a tree of valence at least 3, with the metric in which every edge has length 1?

(6) Find an example of a coarse embedding that is not a quasi-isometric embedding. Find an example of a coarse equivalence that is not a quasi-isometry.

(7) Find an example of quasi-isometric spaces that are not bi-Lipschitz equivalent.

(8) Show that the following spaces are not quasi-isometric one to another (with their natural metrics). The line $\mathbb{R}$, the Euclidean plane $\mathbb{E}^2$ and the regular tree of valence 4.

(9) Show that the regular tree of valence m is quasi-isometric to the regular tree of valence n (with their natural metrics), for every $n, m > 2$.

(10) Show that two length spaces are quasi-isometric iff they are coarsly equivalent.