Grupy i kompleksy

Damian Osajda
damian.osajda@uwr.edu.pl
http://www.math.uni.wroc.pl/~dosaj/

Ćwiczenia 5

(1) Show that any cyclic subgroup of $\mathbb{Z}^2$ is undistorted. Conclude that $\mathbb{Z}^2$ and $BS(1,2)$ are not quasi-isometric.

(2) Show that the plane $\mathbb{R}^2$ with the river metric is an $\mathbb{R}$–tree.

(3) Show that in an $\mathbb{R}$–tree any two points are connected by a unique topological arc.

(4) Show that if in a metric space X any two points are connected by a unique topological arc then X is an $\mathbb{R}$–tree.

(5) Show that every finite group acting by isometries on a complete $\mathbb{R}$–tree fixes a point.

(6) Show the Helly property for an $\mathbb{R}$–tree: Any family of pairwise intersecting connected sets have an intersection.

(7) Show that every quasi-tree is δ–hyperbolic. How to compute δ?

(8) Show that the Euler characteristic of a triangulated disc is 1.

(9) Show that a hyperbolic triangulation of the plane is not quasi-isometric to a tree.

(10) Show that the 1–skeleton of a tessellation of the plane by n–gons, with $n \geqslant 7$, without degree–2 vertices is hyperbolic.

(11) Show that the 1–skeleton of a tessellation of the plane by squares such that every vertex belongs to at least 5–squares is hyperbolic.

(12) Let α, β be two geodesics in a δ–hyperbolic space (X,d) with $\alpha(0) = \beta(0)$ and $d(\alpha(T), \beta(T)) \leqslant D$. Then, for any $t \in [0,T]$ we have $d(\alpha(t), \beta(t)) \leqslant 2(D+\delta)$.

(13) Show that any geodesic n–gon in a δ–hyperbolic space is $\delta(n-2)$–thin.

(14) Show that the discrete Heisenberg group $H_3(\mathbb{Z})$ is not hyperbolic.

(15) Show that the product of two infinite groups is not hyperbolic.

(16) Show that the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ is not hyperbolic.