

Grupy i kompleksy

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Ćwiczenia 6

- (1) Show that a group is hyperbolic iff it acts geometrically on a hyperbolic space.
- (2) Show that hyperbolic groups are finitely presented.
- (3) (*Quasi-Centres*). Let X be a δ -hyperbolic space and Y its non-empty bounded subspace. Let $r_Y = \inf\{\rho > 0 \mid Y \subseteq B(x, \rho) \text{ for some } x \in X\}$. Show that for every $\epsilon > 0$ the set $\{x \in X \mid Y \subseteq B(x, r_Y + \epsilon)\}$ has diameter less than $4\delta + 2\epsilon$.
- (4) Show that the 1-skeleton of any Rips complex of the vertex set of a finite tree is dismantlable.
- (5) Show that a finite flag simplicial complex whose 1-skeleton is dismantlable is contractible.
- (6) Show that for a finite dismantlable graph there exists a clique fixed by all automorphisms.
- (7) Show that a finite graph is dismantlable iff it is cop-win.
- (8) Show that if Γ is dismantlable then the 1-skeleton of any its Rips complex $P_D(\Gamma^{(0)})$ is also dismantlable.
- (9) Show that on a δ -hyperbolic graph a fast cop can catch an even faster robber. How fast has the cop to be? How fast can the robber be?
- (10) Let $F_2 = \langle a, b \rangle$, $F'_2 = \langle c, d \rangle$ and $G = (F_2 \times F'_2) \rtimes \mathbb{Z}_2$ with $\mathbb{Z}_2 = \langle x \rangle$ acting by x interchanging a and b , and c and d . Let $H = \ker(G \rightarrow \mathbb{Z})$ where the map is induced by $F_2 \times F'_2 \rightarrow \mathbb{Z}$ sending each of a, b, c, d to the generator of \mathbb{Z} . Show that H is finitely generated and has infinitely many conjugacy classes of finite subgroups.

Hint: Observe that $H = K \rtimes \mathbb{Z}_2$.