Grupy i kompleksy

Damian Osajda
damian.osajda@uwr.edu.pl
http://www.math.uni.wroc.pl/∼dosaj/

Ćwiczenia 7

(1) Give an example of a finitely generated subgroup H of a group G such that the inclusion $H \hookrightarrow G$ is a quasi-isometric embedding but H is not quasi-convex in a Cayley graph of G.

(2) Show that a finitely generated subgroup of a finitely generated free group is quasi-convex.

(3) Is the notion of quasi-convexity a quasi-isometry invariant?

(4) Let $\Gamma = \mathrm{Cay}(G, S)$ be a Cayley graph of G for a finite generating set S. Let $H < G$ be a subgroup quasi-convex in Γ. Show that then H is finitely generated and the inclusion $H \hookrightarrow G$ is a quasi-isometric embedding.

(5) Let H be a finitely generated subgroup of a hyperbolic group G. Show that the notion of "quasi-convexity" for H is well defined.

(6) Let $\gamma, \gamma' \colon [0, T] \to X$ be two geodesics in a δ–hyperbolic space with a common start-point $\gamma(0) = \gamma'(0)$. Show that for every $t \in [0, T]$ we have $d_X(\gamma(t), \gamma'(t)) \leqslant 2(\delta + d_X(\gamma(T), \gamma'(T)))$.

(7) Let $C(g) = \langle T \rangle$. Show that $Z(C(g)) = \bigcap_{t \in T} C_{C(g)}(t)$.

(8) Let g be an element of infinite order in a hyperbolic group. Show that if g^p is conjugated to g^q then $|p| = |q|$.