Grupy i kompleksy

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Ćwiczenia 8

Conjecture (Kaplansky's Zero Divisor Conjecture)**.** *Let K be a field and let G be a torsion-free group. Then the group ring $K[G]$ contains no non-trivial zero divisors.*

A proof of the Zero Divisor Conjecture
for some hyperbolic groups by T. Delzant:

Let G be a torsion free group acting geometrically on a δ–hyperbolic graph Γ. Suppose that for every $g \in G\backslash\{1\}$, the *minimal displacement* $|g| = \min\{d(gv, v) \mid v \in V(\Gamma)\} > 22\delta$.

(1) Let $g, h \in G$. Show that for every vertex $v \in V(\Gamma)$ we have either $d(ghv, v) > d(gv, v)$ or $d(gh^{-1}v, v) > d(gv, v)$.
Hint: Proceed by a contradiction:
   (a) Consider a vertex q on a geodesic $[v, hv]$ at distance $\lfloor d(hv, v)/2 \rfloor$ from v. Show that q is at distance at most 8δ from $[g^{-1}v, v]$.
   (b) Consider a vertex q' on a geodesic $[v, h^{-1}v]$ at distance $\lfloor d(h^{-1}v, v)/2 \rfloor$ from v. Show that q' is at distance at most 8δ from $[g^{-1}v, v]$.
   (c) Show that $d(q, q') \leqslant 20\delta$.
   (d) Conclude that there exists a vertex w such that $d(w, hw) \leqslant 22\delta$.

(2) Prove that G satisfies the *Unique Product Property*: Let $A, B \subset G$ be two finite non-singletons, and let $C = AB := \{ab \mid a \in A, b \in B\}$. Then there exists an element $c \in C$, which can be written uniquely as a product $c = ab$.
Hint: Consider an "extremal" element in C and use (1).

(3) Show that $K[G]$ has no non-trivial zero divisors.