Grupy i kompleksy

Damian Osajda
damian.osajda@uwr.edu.pl
http://www.math.uni.wroc.pl/∼dosaj/

Ćwiczenia 9

(1) Show that the product of median graphs is median.

(2) Show that every graph embedding isometrically into a median graph is median. Why is the isometricity assumption needed?

(3) Show that intervals in median graphs are median.

(4) Show that a retract of a median graph is median.

(5) Show that the covering graph of a distributive lattice is median.

(6) Show that a lattice may be defined algebraically as a structure $(L, \vee, \wedge)$, where $\vee$ and $\wedge$ are binary operations on L satisfying the following conditions:
   (a) (*Commutativity*) $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$;
   (b) (*Associativity*) $a \vee (b \vee c) = (a \vee b) \vee c$ and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$;
   (c) (*Absorption*) $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$.

(7) Give an example of a poset not being a lattice, and one of a non-distributive lattice.

(8) Show that a Boolean lattice of subsets is distributive.

(9) Show that intervals in median graphs are covering graphs of distributive lattices.

(10) Show that gated sets satisfy the *finite Helly property*, that is, every family of pairwise intersecting gated sets has a nonempty intersection.

(11) Show that for every set V of vertices of a graph there exists a smallest gated set containing A.

(12) Show that gated sets are convex.

(13) Show that the projection map on a gated set is well defined.

(14) Show that for every edge $\{v, w\}$ ia median graph, the set $\{u \mid d(v, u) < d(w, u)\}$ is gated.

(15) Show that a graph is median iff every its interval is gated.

(16) Show that a gated amalgam of median graphs is median.