Grupy i kompleksy

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Ćwiczenia 10

(1) Let $e = \{v, w\} \in E\Gamma$. Let e_v be the set of vertices u of Γ for which there exists a geodesic between u and v omitting $[e]_{\|}$. Let e_w be the set of vertices u of Γ for which there exists a geodesic between u and w omitting $[e]_{\|}$.

  (a) Let $e' = \{v', w'\} \in E\Gamma$ be such that $d(v', w) > d(v', v)$ and $d(w', v) > d(w', w)$. Then $e' \in [e]_{\|}$.
  (b) Show that if there exists a geodesic between u and v omitting $[e]_{\|}$ then every geodesic between u and v omits $[e]_{\|}$.
  (c) Show that $e_v \cap e_w = \emptyset$ and $e_v \cup e_w = V\Gamma$.
  (d) Show that $e_v = \{u \in V\Gamma \mid d(u, w) > d(u, v)\}$.

(2) Show that the square complex of a median graph is simply connected.

(3) Show that every for every loop in a median graph there is a disc diagram.

(4) Give an optimal bound for the area of the minimal disc diagram for a loop of length n.

(5) Show that a group acting geometrically on a median graph is finitely presented.

(6) Show that in a median graph the widths of bigons are uniformly bounded iff the sizes of isometric square grids are bounded.

(7) Let a group G act geometrically on a median graph Γ. Show that G is hyperbolic iff Γ does not contain an isometrically embedded infinite square grid.