McGill University
Department of Mathematics and Statistics
MATH 599
Winter 2017

Combinatorial negative curvature

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Prerequisite: The participants should be familiar with basic notions of (combinatorial) group theory (Cayley graph, free group), and algebraic topology (covering space, fundamental group, homology groups).

Syllabus:

This course is devoted mainly to exploring questions within the Geometric Group Theory. However, I plan to emphasize also appearances of negative curvature, in particular Gromov hyperbolicity, in other branches of mathematics and computer science. Examples are algorithmic graph theory, and network theory.

Geometric Group Theory studies groups from a geometric viewpoint. In general, it means exploring geometric features of metric spaces on which a given group acts in a "reasonable" way. From this we may conclude various algebraic properties of the group.

The general theme of this course is, widely understood, *combinatorial negative curvature*. This notion refers to combinatorial conditions implying (metric) negative-curvature-like phenomena. An example is Gromov hyperbolicity of discrete metric spaces – it can be characterized locally, by combinatorial conditions, under some weak global assumptions. Other examples are CAT(-1) cubical complexes, and 7–systolic complexes. The ultimate goal of the course is to present a relatively recent result – a construction of high-dimensional hyperbolic Coxeter groups. On the way we will explore basic notions and techniques of Geometric Group Theory, and some of metric and algorithmic graph theory.

The outline of the topics covered is as follows:

- Large scale geometry: coarse embedding, quasi-isometry, coarse invariants of metric spaces;
- Gromov hyperbolicity: hyperbolic spaces, quasi-isometric invariance, hyperbolic groups and their basic properties, dismantlability and its (algorithmic) applications;
- Right-angled Coxeter groups: large simplicial complexes, median graphs, CAT(-1) cubical complexes, Coxeter-Davis complex, residual finiteness, virtual freeness, dimension;
- Construction of high-dimensional hyperbolic right-angled Coxeter groups.

The participants should be familiar with basic notions of (combinatorial) group theory (Cayley graph, free group), and algebraic topology (covering space, fundamental group, homology groups).

References

[BH99] Martin R. Bridson and André Haefliger, *Metric spaces of non-positive curvature*, Vol. 319, Springer-Verlag, Berlin, 1999.

[Dav08] Michael W. Davis, *The geometry and topology of Coxeter groups*, London Mathematical Society Monographs Series, vol. 32, Princeton University Press, Princeton, NJ, 2008.

[DK16] Cornelia Druţu and Michael Kapovich, *Geometric group theory*, 2016, available at https://www.math.ucdavis.edu/ kapovich/EPR/ggt.pdf.

[Geo08] Ross Geoghegan, *Topological methods in group theory*, Graduate Texts in Mathematics, vol. 243, Springer, New York, 2008.

[GdlH] Étienne Ghys and Pierre de la Harpe, *Espaces métriques hyperboliques*, Sur les groupes hyperboliques d'après Mikhael Gromov (Bern, 1988), Progr. Math., vol. 83, Birkhäuser Boston, Boston, MA (French).

[Hat02] Allen Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002.
[NY12] Piotr Nowak and Guoliang Yu, *Large scale geometry*, EMS Textbooks in Mathematics, European Mathematical Society (EMS), Zürich, 2012.
[Osa13] Damian Osajda, *A construction of hyperbolic Coxeter groups*, Comment. Math. Helv. **88** (2013), no. 2, 353–367.

Assessement: The evaluation will be based on a presentation of a topic related to the course given by a student.

Academic Integrity: McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/integrity/ for more information).

Language Policy: In accord with McGill Universitys Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Extraordinary Circumstances: In the event of extraordinary circumstances beyond the Universitys control, the content and/or evaluation scheme in this course is subject to change.