

## Combinatorial negative curvature

Damian Osajda

Problem list 3

- (1) Show that any geodesic  $n$ -gon in a  $\delta$ -hyperbolic space is  $\delta(n-2)$ -thin.
- (2) Show that quasi-trees are hyperbolic. What is the hyperbolicity constant?
- (3) Show that the direct product of a tree and a bounded diameter graph is a quasi-tree.
- (4) Show that the direct product of a hyperbolic graph and a bounded diameter graph is hyperbolic. What is the hyperbolicity constant?
- (5) Show that the product of two unbounded graphs is not hyperbolic.
- (6) Prove the Combinatorial Gauss-Bonnet Formula.
- (7) Show that the 1-skeleton of a tessellation of the plane by  $n$ -gons, with  $n \geq 7$ , without degree-2 vertices is hyperbolic.
- (8) Show that the 1-skeleton of a tessellation of the plane by squares such that every vertex belongs to at least 5-squares is hyperbolic.
- (9) Show that a geodesic space is discretely geodesic.
- (10) Show that for any discretely geodesic space  $X$  there exists  $D > 0$  such that the Rips graph  $P_D(X)$  is geodesic.
- (11) Formulate the Morse Lemma for discretely geodesic spaces. Prove it by modifying the proof from the lecture.
- (12) Show that the discrete Heisenberg group  $H_3(\mathbb{Z})$  is not hyperbolic.
- (13) Show that the Baumslag-Solitar group  $BS(1, 2)$  is not hyperbolic.
- (14) Show that hyperbolic groups are finitely presented.
- (15) Let  $X$  be a discretely geodesic  $\delta$ -hyperbolic metric space. Show that:
  - (a) There exists  $\delta'$  such that every triangle has a  $\delta'$ -centre;
  - (b) (Four point condition) There exists  $\delta''$  such that  $d(v, u) + d(w, z) \leq \max\{d(v, z) + d(u, w), d(v, w) + d(u, z)\} + \delta''$ , for any four points  $v, u, w, z$ .