

Combinatorial negative curvature

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Problem list 3

- (1) Show that any geodesic n -gon in a δ -hyperbolic space is $\delta(n-2)$ -thin.
- (2) Show that quasi-trees are hyperbolic. What is the hyperbolicity constant?
- (3) Show that the direct product of a tree and a bounded diameter graph is a quasi-tree.
- (4) Show that the direct product of a hyperbolic graph and a bounded diameter graph is hyperbolic. What is the hyperbolicity constant?
- (5) Show that the product of two unbounded graphs is not hyperbolic.
- (6) Prove the Combinatorial Gauss-Bonet Formula.
- (7) Show that the 1-skeleton of a tessellation of the plane by n -gons, with $n \geq 7$, without degree-2 vertices is hyperbolic.
- (8) Show that the 1-skeleton of a tessellation of the plane by squares such that every vertex belongs to at least 5-squares is hyperbolic.
- (9) Show that a geodesic space is discretely geodesic.
- (10) Show that for any discretely geodesic space X there exists $D > 0$ such that the Rips graph $P_D(X)$ is geodesic.
- (11) Formulate the Morse Lemma for discretely geodesic spaces. Prove it by modifying the proof from the lecture.
- (12) Show that the discrete Heisenberg group $H_3(\mathbb{Z})$ is not hyperbolic.
- (13) Show that the Baumslag-Solitar group $BS(1, 2)$ is not hyperbolic.
- (14) Show that hyperbolic groups are finitely presented.
- (15) Let X be a discretely geodesic δ -hyperbolic metric space. Show that:
 - (a) There exists δ' such that every triangle has a δ' -centre;
 - (b) (Four point condition) There exists δ'' such that $d(v, u) + d(w, z) \leq \max\{d(v, z) + d(u, w), d(v, w) + d(u, z)\} + \delta''$, for any four points v, u, w, z .