

## Combinatorial negative curvature

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Problem list 4

- (1) Show that the flag completion of a dismantlable graph is contractible.
- (2) Let  $v$  be a dominated vertex of a dismantlable graph  $\Gamma$ . Show that the subgraph of  $\Gamma$  induced by  $V(\Gamma) \setminus \{v\}$  is dismantlable.
- (3) Show that a finite dismantlable graph contains a clique fixed (set-wise) by all its automorphisms.
- (4) Show that every quasi-centre of a bounded subspace  $Y$  of a hyperbolic space is fixed (set-wise) by all automorphisms fixing (set-wise)  $Y$ .
- (5) Let a finite group  $H$  act by automorphisms on a tree  $T$ . Show that there exists a finite  $H$ -invariant dismantlable subgraph of  $T$ .
- (6) Show that a graph is dismantlable iff it is cop-win.
- (7) Show that each Rips graph of a dismantlable graph is dismantlable.
- (8) Show that the one-skeleton of the barycentric subdivision of the flag completion of a dismantlable graph is dismantlable.
- (9) Show that  $\mathcal{CWFR}(s, s') \subseteq \mathcal{CWFR}(s, s'')$ , for  $s'' > s'$ .
- (10) Show that for every  $s' \leq s$ , a graph  $\Gamma$  is in  $\mathcal{CWFR}(s, s')$  iff  $\Gamma$  is  $(s, s')$ -dismantlable.